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An Analysis and Comparison of the Common Core State Standards for Mathematics and the Singapore Mathematics Curriculum Framework

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AN ANALYSIS AND COMPARISON OF THE COMMON CORE STATE STANDARDS FOR MATHEMATICS AND THE SINGAPORE MATHEMATICS CURRICULUM FRAMEWORK

by

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ABSTRACT

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In this analysis and comparison we look at the Common Core State Standards for Mathematics and the Singapore Mathematics Curriculum Framework, standards documents that guide primary and secondary mathematics education in the United States and Singapore respectively. The official Common Core State Standards for Mathematics website claims that the standards have been developed to be "internationally benchmarked, so that all students are prepared for the 21st century". Singapore has recently been recognized as a world leader in mathematics education. We investigate the claim that the Common Core State Standards for Mathematics are internationally benchmarked by comparing the Common Core State Standards for Mathematics to the Singapore Mathematics Curriculum Framework.

We first give a brief overview of both mathematics standards documents. Then we proceed to determine the alignment of the two sets of standards, both in terms of the coverage of mathematics topics and levels of cognitive demand, using the Surveys of Enacted Curriculum content analysis method. We find that the two standards documents are similar in terms of content coverage, but that the Common Core State Standards for

Mathematics exhibit higher percentages of standards that require higher levels of cognitive demand.

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CHAPTER 1: COMMON CORE STATE STANDARDS

1.1 Background and Structure of Common Core State Standards for Mathematics

The Common Core State Standards (CCSS), released on June 2, 2010, are a set of English language arts and mathematics standards for grades kindergarten through twelve, commissioned by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA). The Common Core State Standards website provides the following mission statement [3]:

> The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy.

The Common Core State Standards are not national or federal standards, but rather a set of standards that may be voluntarily adopted by each state. According to the official Common Core website forty-four states, four territories (American Samoa Islands, Guam, Northern Mariana Islands, and U.S. Virgin Islands), the District of Columbia, and the Department of Defense Education Activity have all adopted these standards. Alaska, Minnesota, Nebraska, Puerto Rico, Texas, and Virginia all have yet to fully adopt the standards. Kentucky was the first state to fully implement the standards during the 2011-2012 academic year, while other states will not fully implement the standards until the 2015-2016 academic year. In March 2014, Indiana became the first state to withdraw from the CCSS. Although they have withdrawn, the newly drafted version of their state's standards still draws heavily from the standards present in the CCSS [4].

The Common Core State Standards for Mathematics (CCSSM) are organized in two groups: Standards for Mathematical Practice (also referred to as the Math Practice Standards or practice standards) and Standards for Mathematical Content. The Standards for Mathematical Practice describe ways of thinking about mathematics that are to be developed in order for students to become mathematically proficient. The practice standards were developed based on the National Council of Teachers of Mathematics (NCTM) process standards and the strands of mathematical proficiency as published in *Adding It Up* by the National Research Council. The Math Practice Standards included in the CCSSM are the following [20]:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Standards for Mathematical Practice are infused throughout the Standards for Mathematical Content for all grades.

The Standards for Mathematical Content are designed to be a balance of standards involving procedures and understanding. For grades kindergarten through eight, content standards are organized explicitly by individual grade levels. Then, within each individual grade level, the standards are organized into domains and further divided into subdomains referred to as clusters. Content standards for high school are not organized by individual grade levels, but are instead organized into the following six conceptual categories that are intended to span all of grades nine through twelve [20]:

- Number and quantity,
- Algebra,
- Functions,
- Modeling,
- Geometry, and
- Statistics and probability.

In addition to these conceptual categories, the content standards also include additional mathematics that is indicated by a (+) symbol. This symbol designates that the content of these standards is not required of all students but is important if students plan to take advanced mathematics courses such as calculus, advanced statistics, or discrete mathematics. Another symbol that is used in the standards is a star symbol (★); this symbol indicates that the preceding standard, or group of standards, involves mathematical modeling. Authors of the standards feel that mathematical modeling is best viewed in relation to other mathematics topics, and thus standards involving modeling have been included throughout the high school standards.

1.2 Motivation

In a video by The Hunt Institute titled, “The Mathematics Standards: How They Were Developed and Who Was Involved”, two of the standards writers William McCallum and Jason Zimba give clear explanations regarding the motivation for the standards document [9]. In the video William McCallum explains that [9]

> We had a situation where different states had widely different standards. The curriculum in the United States has often been criticized for being ‘a mile wide and an inch deep’, and we had an opportunity to really write something aspirational, something that brought states together around a common understanding of what we wanted to get out of our education system and what we wanted our children to know.

According to Jason Zimba, the intent was not for each and every state to begin teaching mathematics in exactly the same way, but to collaborate and work to improve on what the most successful states were doing. Zimba claimed that [9]

> In producing these standards, the working group was charged with using evidence to an unprecedented degree; evidence about what high performing countries do in mathematics, evidence about the true demands of college and careers.

In a second video by the Hunt Institute titled “The Mathematics Standards: Key Changes and Their Evidence”, McCallum and Zimba also specifically mentioned that in writing the standards they carefully examined the standards of high-achieving East Asian countries such as Hong Kong, Japan, Korea, and Singapore [10].

In the first video McCallum and Zimba emphasize that, in creating the CCSSM, their main goal was to create a set of standards that are both focused and coherent. Coherent in the sense that as students advance from one grade level to the next there would be continuity and clear expectations of what students had previously learned and what they would be learning in the next grade level as well. This way teachers would have a clear understanding of how the portion of the curriculum they were teaching fit into the curriculum as a whole. The second guiding principle for the writing team was focus. In the Hunt Institute video “The Mathematics Standards: How They Were Developed and Who Was Involved”, Zimba explains [9]

> Focus means spending more time on fewer things at any given grade, principally on number and operations in early grades. This is to give teachers more time to teach those things to mastery and give students a firm foundation on which to

> build. And the point is that math is not like a homogeneous fluid that can be ladled into bowls and served to students. It has a logical structure with lots of connections, some of them intricate.

1.3 Authors of the Mathematics Standards

The Common Core State Standards for Mathematics were principally written by a central team consisting of three individuals: William (Bill) McCallum, Jason Zimba, and Philip (Phil) Daro. William McCallum received his Ph.D. in mathematics from Harvard University in 1984, and currently he is a University Distinguished Professor of Mathematics and Head of the Department of Mathematics at the University of Arizona. Jason Zimba received his Ph.D. in physics from the University of California-Berkeley in 2001, and has taught university level physics and mathematics courses. Philip Daro is a mathematics educator and has directed large-scale teacher professional development programs, and currently he is the Site Director of the Strategic Education Research Partnership (SERP) at the San Francisco Unified School District. All three team members have also been members on various mathematical committees and have had influential roles in the field of mathematics education.

Although the CCSSM were written primarily by the members of the central team, the team also worked with professional organizations and groups of experts in the fields of mathematics and mathematics education including the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), the Association of Mathematics Teacher Educators (AMTE), the American Mathematical Society (AMS), the Mathematical Association of America (MAA), state mathematics directors, mathematicians, education researchers, teachers, and policymakers. According to Dr.

McCallum, a work team of about 60 people, all from the aforementioned groups and organizations, worked together to provide feedback to the central team [9]. In addition, drafts were sent out to the individual states and opportunities for feedback were given. After rounds of revision the final standards document was released in June 2010.

1.4 The Strands of Mathematical Proficiency

The strands of mathematical proficiency, as published in *Adding It Up: Helping Children Learn Mathematics* by the National Research Council, are five interwoven components, or strands, that the authors claim are essential to learning mathematics. The strands were developed by the Committee on Mathematics Learning which was created by the National Research Council in 1998. The committee was sponsored by the National Science Foundation and the United States Department of Education. In developing the strands, the committee's goal was to, "provide a framework for discussing the knowledge, skills, abilities, and beliefs that constitute mathematical proficiency" [22]. The five strands, as seen in Figure 1 below, are [22]:

i) *Conceptual understanding,* comprehension of mathematical concepts, operations, and relations;

ii) *Procedural fluency,* skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;

iii) *Strategic competence,* ability to formulate, represent, and solve mathematical problems;

iv) *Adaptive reasoning,* capacity for logical thought, reflection, explanation, and justification;

v) *Productive disposition,* habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

The authors strongly emphasize that "the five strands are interwoven and interdependent in the development of proficiency in mathematics" [22]. In other words, as the analogy of the tightly bound rope suggests, one or two strands by themselves are not enough to define mathematical proficiency; all of the strands play an equally important role.

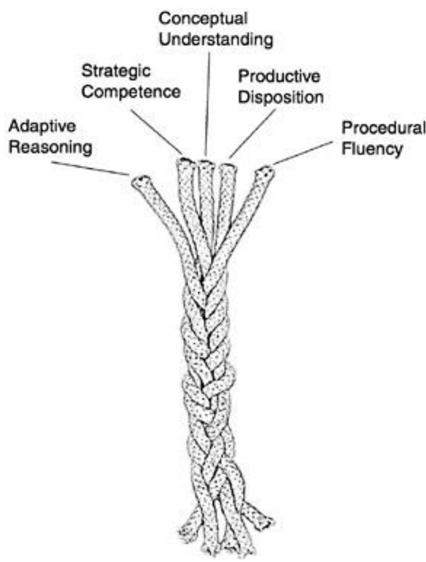


Figure 1: The Strands of Mathematical Proficiency [22]

1.5 Authors of the Strands

The Committee on Mathematics Learning, a 16 member committee, included individuals from various backgrounds, with the majority of members being current university professors in either the field of mathematics or mathematics education (refer to Appendix A for the names and background information of individual members of the committee). The committee worked collaboratively over the course of an 18-month period to write *Adding It Up: Helping Children Learn Mathematics* in which one of the introductory chapters of the text is devoted to the strands [22].

CHAPTER 2: SINGAPORE MATHEMATICS CURRICULUM FRAMEWORK

2.1 Brief Overview of Singapore's Education System

Singapore is a small island country, approximately 275 square miles in total area, located just off the Malay Peninsula in Southeast Asia. The population of Singapore is 5.3 million, comparable in population to the state of Colorado but less than a quarter of the geographic size of the state of Rhode Island. Singapore has four official languages: English, Malay, Chinese, and Tamil, and its major ethnic groups are: Chinese (75.2%), Malay (13.6%), Indian (8.8%), and Other (2.4%) [6].

The education system in Singapore is highly centralized. It is organized by the Ministry of Education (MOE) whose current minister, Mr. Heng Swee Keat, was appointed on May 21, 2011 [8]. The MOE has developed a national curriculum, centered on a detailed syllabus that is meant to help guide teachers in both planning and implementing successful mathematics programs. Teachers are given the freedom to be innovative in their presentation of the material within their own classrooms but are also responsible for ensuring that the curriculum prepares students for high-stakes national examinations at the end of both primary and secondary school [13].

Having been a former British colony, Singapore's education system is based on the traditional British education system. The Singaporean system, as represented below in Figure 2 [7], is flexible in structure so that it meets the needs of individual students. For most students their education begins at age four with two years of private kindergarten. From here, students enter the national school system (typically at age six) where they will complete a total of six years of primary (elementary) school, broken into two stages, with the first four years of primary schooling comprising the foundation stage and the final two years the orientation stage. During the foundation stage, 80% of the curriculum focuses on English, each student's mother tongue language, and mathematics. Starting with primary grade three, science is introduced into the curriculum. In 2008, subject-based banding was introduced to replace streaming that had been in place for primary grades five and six [6]. Subject-based banding allows students to take courses at either the standard or foundation level based on their individual strengths. As an example, if a student is strong in mathematics and science but weak in English and mother tongue, that student would take mathematics and science at the standard level and English and mother tongue at the foundation level. If students are successful in subjects at the foundation level they may be allowed to transition to the standard level for primary grade six. At the end of the sixth year of primary school students take the Primary School Leaving Examination (PSLE), a rigorous exam that tests students' abilities in English, mathematics, science, and their mother tongue language.

Based on each student's individual score on the PSLE, he or she begins secondary school following one of three paths, formally called streams, including [15]:

i) four years in the Normal (Technical) [N(T)] course,

ii) four years in the Special/Express course, or

iii) five years in the Normal (Academic) [N(A)] course.

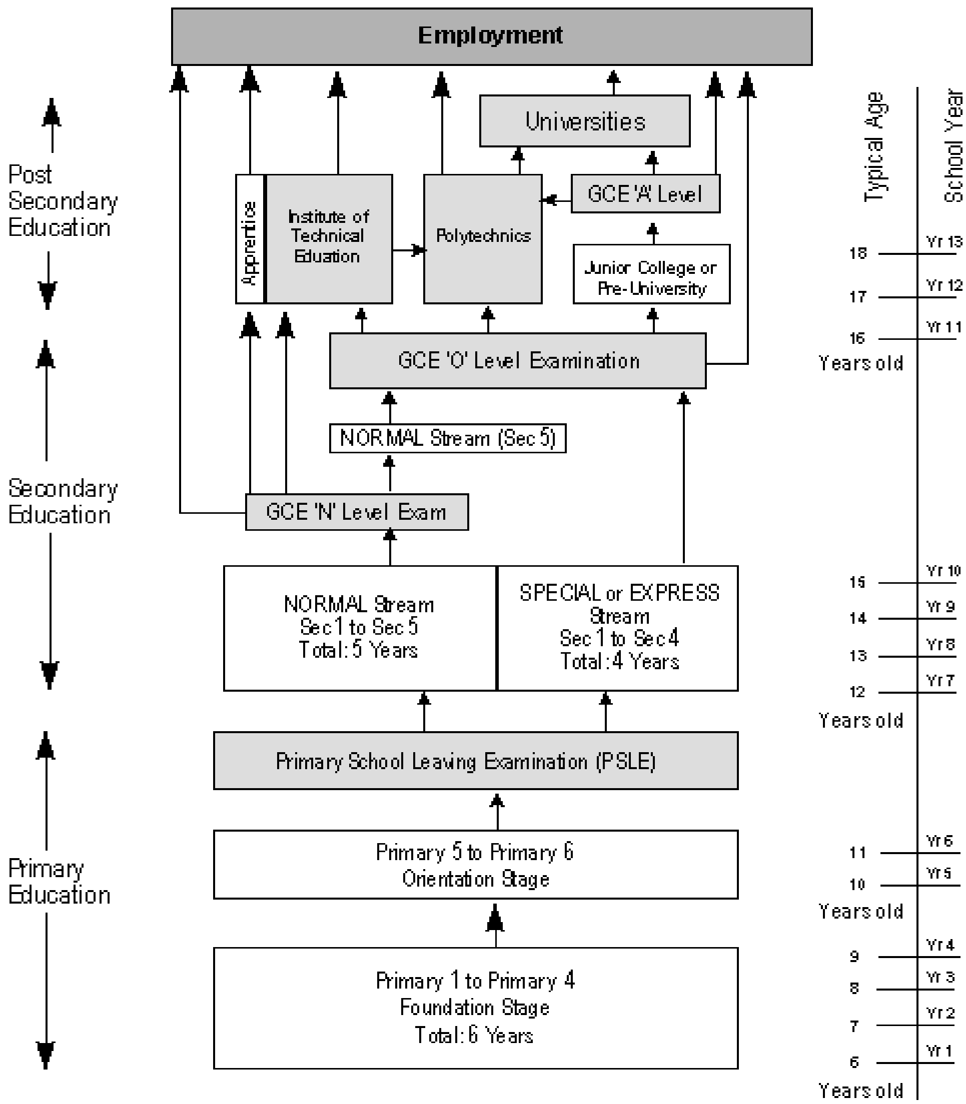


Figure 2: Structure of the Singapore Education System [7]

In addition to these streams, there are also students who are part of the Integrated Programme (IP) which is an enrichment program for high ability students. Students in this program complete six years of secondary education, bypassing the Singapore-Cambridge General Certificate of Education Ordinary (GCE O-Level) Exam, instead sitting for the GCE Advanced (A) Level Examination (university entrance exam).

The Normal (Technical) course accommodates 12.7 percent of students and is for those who need additional academic support [6]. Students in this course learn all of the material included in the national curriculum including mathematics topics such as: graphs of quadratic functions and their properties, rotational symmetry, and the volume and surface area of pyramids, cones, and spheres. After students complete the Normal (Technical) course they take the joint Singapore-Cambridge General Certificate of Education Normal (GCE N-level) examination.

The Special/Express and the Normal (Academic) courses accommodate 61.8 percent and 25.5 percent of students respectively [6]. Both of these courses are similar in content; the main difference is the length of study required to complete each course, four years to complete the Special/Express course and five years to complete the Normal (Academic) course. Mathematics topics include: integers, real numbers, Cartesian geometry, algebraic equations and graphs, Pythagoras theorem, trigonometry, circle properties, transformation geometry, and statistics and probability. Following their fourth year, students in the Special/Express course complete the GCE O-Level college entrance examination. Similar to students in the Normal (Technical) course, students in the Normal (Academic) course complete the N-Level exam after their fourth year. Then, after an additional fifth year of schooling, depending on their performance on the N-level exam, students may be able to take the O-Level exam as well. As noted, at each level of education, students must pass a rigorous examination in order to begin the next academic stage.

As seen in Figure 3 below [19], students have flexibility to move from one course to another, depending on their performance and evaluation from teachers and principals.

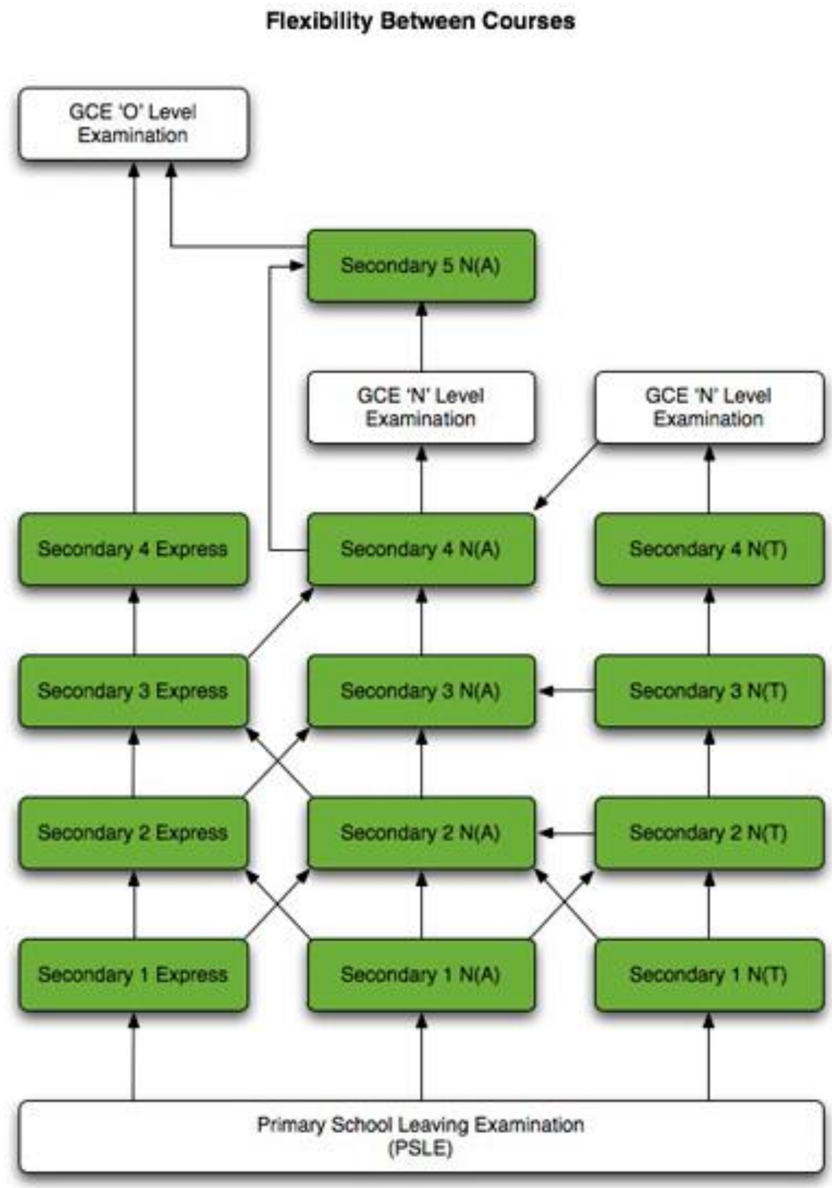


Figure 3: Course Flexibility in Singaporean Secondary Education [19]

Following the completion of secondary school (the United States equivalent to grade ten), education is no longer compulsory. However, in a recent study it was found that in 1999, 78% of Singaporean students attended post-secondary educational institutions, and in 2011 that number had risen even higher to 93% [13]. In comparison, in the United States, the compulsory school attendance age varies by state ranging from age 16 to 18. In 2012, 71.3% of recent U.S. high school graduates were enrolled in either a 2-or 4-year college [1].

In Singapore, for students who do choose to pursue post-secondary education, there are many paths for students that will lead to technical institutes, junior colleges, or universities and ultimately result in future employment.

2.2 Structure of Singapore Mathematics Curriculum Framework

The earliest evidence of the development of a mathematics syllabus in Singapore dates back to 1957 [5]. This syllabus was called Syllabus B and was for secondary education. In the 1970's an additional syllabus, Elementary Mathematics (Syllabus C), was introduced; this syllabus included modern mathematics topics such as commutative and associative laws, sets, transformation geometry, and vectors. Then in the early 1980's Elementary Mathematics (Syllabus D) was also created which was provided a balance of traditional and modern topics [6]. In the years following flaws began to surface and educational initiatives were introduced that helped to shape the syllabi into the documents they are today. The documents comprising the syllabi are known as the Singapore Mathematics Curriculum Framework (SMCF); they include syllabi for both primary and secondary education. The SMCF previously consisted of two syllabi, both published in 2007, the first titled Mathematics Syllabus Primary for primary grades one through six and the second titled Secondary Mathematics Syllabuses for secondary grades seven through ten. In 2013, the syllabi underwent revisions, from which emerged three new documents, the Primary Mathematics Teaching and Learning Syllabus, O-& N(A)-Level Mathematics Teaching and Learning Syllabus and N(T) -Level Mathematics Teaching and Learning Syllabus. These syllabi are currently being implemented year-by-year to be fully implemented by 2016 [18].

At the minimum level, in terms of content coverage, students in Singapore are expected to meet the requirements of the N(T)-Level Syllabus, but because the majority (approximately 87%) of students in Singapore complete either the Special/Express (leading to the O-Level exam) or the N(A) course as the minimum requirement, I will be focusing this analysis on the O- & N(A)-Level Mathematics Teaching and Learning Syllabus [6]. This document begins with an introduction that summarizes the necessity for learning mathematics. The writers claim that [18],

> At the individual level, mathematics underpins many aspects of our everyday activities, from making sense of information in the newspaper to making informed decisions about personal finances. It supports learning in many fields of study, whether it is in the sciences or in business. A good understanding of basics mathematics is essential whenever calculations, measurements, graphical interpretations and statistical analysis are necessary. The learning of mathematics also provides an excellent vehicle to train the mind, and to develop the capacity to think logically, abstractly, critically and creatively. These are important 21st century competencies that we must imbue in our students so that they can lead a productive life and be life-long learners.

The O- & N(A)-Level Mathematics Teaching and Learning Syllabus is then further divided into five chapters [18]:

1) Introduction,
2) Mathematics Framework,
3) Teaching, Learning, and Assessment,
4) O-Level Mathematics Syllabus, and
5) N(A)-Level Mathematics Syllabus.

In the Introduction, the detailed implementation timeline is included [18]:

Year	2013	2014	2015	2016
Level	Secondary 1	Secondary 2	Secondary 3	Secondary 4

Explicit goals and aims are also given in the Introduction. The syllabus states that the broad aims of mathematics education in Singapore are to enable students to [18]:

- acquire and apply mathematical concepts and skills;
- develop cognitive and metacognitive skills through a mathematical approach to problem solving; and
- develop positive attitudes toward mathematics.

Chapter Two of the syllabus provides a pentagonal mathematics framework which is a key feature of the SMCF. This framework, first published in 1990, has mathematical problem solving as its central focus, as seen below in Figure 4 [18]. The five surrounding inter-related components of the framework include: concepts, skills, processes, attitudes, and metacognition. For each of the components included in the framework a detailed explanation is provided [18]:

- *Mathematical concepts* can be broadly grouped in numerical, algebraic, geometrical, statistical, probabilistic, and analytical concepts.
- *Mathematical skills* refer to numerical calculation, algebraic manipulation, spatial visualization, data analysis, measurement, use of mathematical tools, and estimation.
- *Mathematical processes* refer to the process skills involved in the process of acquiring and applying mathematical knowledge. This includes reasoning,

communication and connections, application and modeling, and thinking skills and heuristics that are important in mathematics and beyond.

- *Metacognition*, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, particularly in the selection and use of problem-solving strategies. It includes monitoring of one's own thinking, and self-regulation of learning.
- *Attitudes* refers to the affective aspects of mathematics learning such as:
  - beliefs about mathematics and its usefulness;
  - interest and enjoyment in learning mathematics;
  - appreciation of the beauty and power of mathematics;
  - confidence in using mathematics; and
  - perseverance in solving a problem.

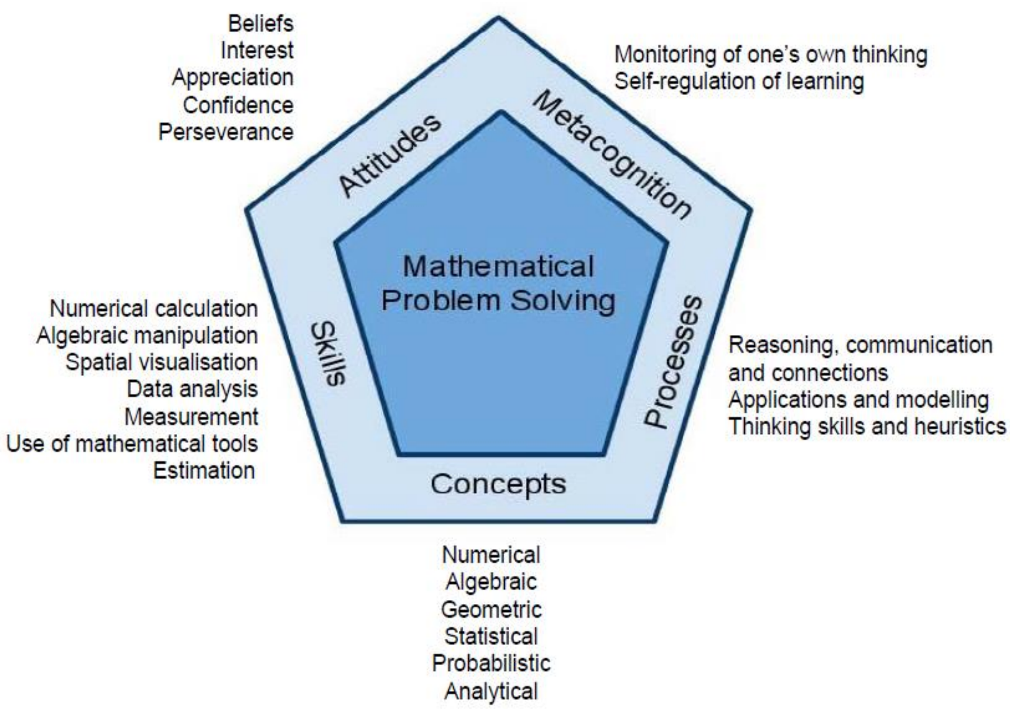


Figure 4: Pentagonal Structure of Singapore Mathematics Curriculum Framework, 2013 [18]

Chapter Three of the syllabus outlines principles for teaching and learning mathematics. Three principles for teaching mathematics are included in this chapter along with a discussion of the role of assessment in the classroom.

Chapters Four and Five, the O-Level Mathematics Syllabus and the N(A)-Level Mathematics Syllabus respectively, detail the mathematics standards that are compulsory for those education levels. Each syllabus is organized along three content strands including: number and algebra, geometry and measurement, and statistics and probability, and one process strand that flows across the content strands. Within each academic course the mathematical topics and subtopics are listed in tables along with the corresponding learning experiences and opportunities that will facilitate the learning of the required content. Figure 5 [18] below shows a sample section which details the organization of the framework as laid out in the O-Level Mathematics Syllabus Secondary One.

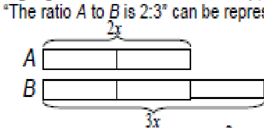
Content	Learning Experiences
Secondary One	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N1. Numbers and their operations	
1.1. primes and prime factorisation 1.2. finding highest common factor (HCF) and lowest common multiple (LCM), squares, cubes, square roots and cube roots by prime factorisation 1.3. negative numbers, integers, rational numbers, real numbers and their four operations 1.4. calculations with calculator 1.5. representation and ordering of numbers on the number line 1.6. use of <, >, ≤, ≥ 1.7. approximation and estimation (including rounding off numbers to a required number of decimal places or significant figures, and estimating the results of computation)	(a) Classify whole numbers based on their number of factors and explain why 0 and 1 are not primes. (b) Discuss examples of negative numbers in the real world. (c) Represent integers, rational numbers and real numbers on the number line as extension of whole numbers, fractions and decimals respectively. (d) Use algebra discs or the AlgeDisc™ application in AlgeTools™ to make sense of addition, subtraction and multiplication involving negative integers and develop proficiency in the 4 operations of integers. (e) Work in groups to estimate quantities (numbers and measures) in a variety of contexts, compare the estimates and share the estimation strategies. (f) Compare follow-through errors arising from intermediate values that are rounded to different degrees of accuracy. (g) Make estimates and check the reasonableness of answers obtained by calculators.
N2. Ratio and proportion	
2.1. ratios involving rational numbers 2.2. writing a ratio in its simplest form 2.3. problems involving ratio	(a) Discuss and explain how ratios are used in everyday life. (b) Use the concept of equivalent ratios to find the ratio a:b:c given the ratios a:b and b:c. (c) Make connections between ratios and fractions, use appropriate mathematical language to describe the relationship, and use algebra to solve problems, e.g. "The ratio *A* to *B* is 2:3" can be represented as:  The ratio 2:3 means "2 units to 3 units", "*A* is $\frac{2}{3}$ of *B*", or "*B* is $\frac{3}{2}$ of *A*". (d) Use the AlgeBar™ application in AlgeTools™ to formulate linear equations to solve problems. (Students can draw models to help them formulate equations.)

Figure 5: Sample Content O-Level Mathematics Syllabus Secondary One [18]

It is important to note that currently the O- & N(A)-Level Mathematics Teaching and Learning Syllabus only contains content standards for secondary grades one and two. The syllabus document explicitly states that secondary grades three and four will be updated accordingly [18]. So for this reason, I have chosen to focus my content strand comparisons and analyses for secondary grades based on the Singapore GCE Mathematics Ordinary Level Syllabus; this syllabus includes mathematics standards for students preparing for the O-Level Exam [23]. Most of the standards in this document are exactly the same as those in the O- & N(A)-Level Mathematics Teaching and Learning

Syllabus, but because the secondary three and four standards were not available for comparison we felt it important to note this distinction.

2.3 Motivation

When Singapore gained its independence from Malaysia in August 1965, it was a poor country that lacked natural resources such as oil and gas. In order to exist independently and survive economically, Singaporeans had to act quickly; their mission involved the development of a strong workforce which they hoped would attract large multi-national companies that would establish plants in Singapore [2]. However, at the time of its independence, Singapore had a 40% illiteracy rate and high unemployment [21].

As a response to this, schools were built quickly, in a so-called “cookie cutter” fashion and teachers were recruited feverishly, often times being recruited from the testing hall directly following the completion of an O-level exam [2]. Teachers that were recruited in this manner taught a full class load, and then spent additional hours in the evening in a teacher training program.

By the late 1970’s some flaws in Singapore’s education system were recognized and measures were taken to improve the system. National exams were instituted for students at age 12, 16, and 18. In 1980, streaming was introduced, and the teacher education system was improved [6]. Curriculum and teaching resources were standardized across all schools. In addition, in 1997 the MOE instituted three educational initiatives: Thinking Schools and Learning Nation (TSLN), Information Technology (IT), and National Education (NE) [5].

Today, according to the former Minister of Education Ng Eng Hen, Singapore is transitioning from strong localized control of education to a more flexible system that allows schools to experiment and work to develop their individual strengths [2].

2.4 Authors of the Singapore Mathematics Curriculum Framework

The Singapore Mathematics Curriculum Framework was created and implemented by the Singapore Ministry of Education (MOE). Specifically, within the MOE, the Curriculum Planning and Development Division (CPDD) is responsible for designing, developing, and monitoring the implementation of syllabi. Within the CPDD there is a Mathematics Unit which works exclusively with the mathematics syllabi. Members of the unit are specialists with a minimum of a master's degree, although some members hold more advanced degrees in either mathematics or mathematics education.

Currently, the MOE consists of four political heads and a senior management team. Appointed by the Prime Minister of Singapore on May 21, 2011, Mr. Heng Swee Keat is serving as the current Minister for Education. The Minister for Education is also a member of the Cabinet of Singapore which forms the executive branch of government along with the President of Singapore [8].

According to their official website, the MOE mission statement includes [17]:

> The wealth of a nation lies in its people - their commitment to country and community, their willingness to strive and persevere, their ability to think, achieve and excel. Our future depends on our continually renewing and regenerating our leadership and citizenry, building upon the experience of the past, learning from the circumstances of the present, and preparing for the challenges of the future. How we bring up our young at home and teach them in school will shape Singapore in the next generation.

CHAPTER 3: METHODOLOGY

3.1 The SMCF Pentagon Model and the Strands of Mathematical Proficiency

To compare the SMCF pentagon model and the strands of mathematical proficiency, I first investigate each framework in terms of structure. Then I identify correspondences between individual components. Finally, I make connections between the corresponding components and elaborate on these connections and what they mean in terms of the CCSSM and the SMCF.

3.2 Overview of Surveys of Enacted Curriculum

The primary tool I chose to use for the content analysis of the CCSSM and the SMCF was the Surveys of Enacted Curriculum (SEC) method of analysis. The SEC was developed in 1998 by a collaborative of state education specialists and researchers at the Wisconsin Center for Education Research (WCER). Rolf Blank, director of education indicators at the Council of Chief State School Officers (CCSSO), led the collaborative, however much of the design and content of the survey was based on research led by former WCER Director Andrew Porter and WCER researcher John Smithson [24].

The SEC is a two-dimensional method of analysis designed to help educators align curriculum, instruction, and assessment. The SEC content analysis procedure has also been recognized for its usefulness in comparing standards documents [11]. The SEC K-12 mathematics taxonomy, as seen in Appendix B [25], contains 16 broad mathematics topics [25]:

1) Number sense/Properties/Relationships;
2) Operations;
3) Measurement;

4) Consumer Applications;
5) Basic Algebra;
6) Advanced Algebra;
7) Geometric Concepts;
8) Advanced Geometry;
9) Data Displays;
10) Statistics;
11) Probability;
12) Analysis;
13) Trigonometry;
14) Special Topics;
15) Functions;
16) Instructional Technology.

Each broad mathematics topic is further divided into anywhere from four to nineteen subtopics (as seen in Appendix C [25]) with each broad topic containing an “other” category for material that does not align with subtopics in any given category. There are a total of 217 topics and subtopics in mathematics. Each topic and subtopic within the taxonomy is assigned a three or four digit code with the exception of “0” which is reserved to code standards that fit into all mathematics topics.

Then, in addition to the topics, each of the standards is also coded with a letter (B-F or Z) based on the level of cognitive demand that it requires. Categories of cognitive demand are [25]:

B) Memorize;

C) Perform Procedures;

D) Demonstrate Understanding;

E) Conjecture/Analyze;

F) Solve Non-Routine Problems;

Z) Non-Specific Cognitive Demand.

The categories of cognitive demand in the SEC taxonomy share the following correspondence with the learning objectives of Bloom's Taxonomy [16]:

Bloom's Taxonomy	**SEC Taxonomy**
Knowledge	Memorize Facts, Definitions & Formulas
Comprehension	Conduct Investigations/ Perform Procedures
Application & Analysis	Communicate Understanding
Synthesis	Analyze Information
Evaluation	Apply Concepts/Make Connections

The WCER has also provided a table (as seen in Appendix D [25]) that provides specific examples of skills and procedures for each category of cognitive demand. According to the SEC coding procedures, a single standard may be coded a maximum of six times (i.e., six combinations of topic and cognitive demand) [25].

3.3 The Mathematics Content Standards

To compare the mathematics standards, I use several methods. First, I include two alignment tables (as seen in Appendices E and F) where I carefully examine each standard within the number and algebra and geometry content strands of the Singapore

GCE O-Level Syllabus and the analogous conceptual categories of the CCSSM. In the left column of each table are all of the standards in the Singapore GCE O-Level Syllabus, and in the right column are the corresponding standards from the CCSSM. In the case where I was unable to locate an analogous standard in the CCSSM I have left the corresponding row entry blank. This table was useful in determining the topical alignment of the two standards documents for all secondary grades.

Next, I calculate the total number of content standards per domain for grades seven and eight (equivalent to secondary one and two, respectively). This again gives me a sense of how the standards documents were divided topically among these individual grades.

Finally, I used the SEC content analysis method to code each individual standard within the CCSSM and O- & N(A)-Level Mathematics Teaching and Learning Syllabus (CCSSM grades seven and eight and SMCF secondary one and two, respectively) and then I proceeded similarly to code each standard of the CCSSM and GCE Mathematics O- Level Secondary Syllabus for the respective conceptual categories and content strands number and algebra and geometry. Based on the SEC taxonomy, each standard was coded with a three or four digit number and a letter. For example, SCMF secondary two standard N.7.7 requires that students “solve simultaneous linear equations in two variables” [18]. I coded this standard as 602C. The code 602 refers to the subtopic *systems of equations* which is under the mathematics topic *advanced algebra*, and the letter C for the cognitive demand category *perform procedures*.

After I coded each standard, based on the intersection of topics and cognitive demand, I compiled the data. First, I compiled the total number of codes for each grade or conceptual category/content strand. I have called this number *n* and placed it in the first cell of each SEC coding table presented in Chapter 4. Then we calculate the total number of codes, per grade level or conceptual category/content strand, formed by the intersection of each topic and cognitive demand category. We take the number of individual codes form each intersection and divide by the total number of codes per grade level or conceptual category/content strand; this ratio represents the percentage of the overall curriculum, per grade level or conceptual category/content strand, that is devoted to each topic and cognitive demand category.

The official SEC website has content analysis results for the CCSSM for each individual primary school grade (kindergarten through eight); for secondary grades content analysis results are not broken down by grade but are given based on all of grades nine through twelve. There are only limited results for the SCMF. It is important to note that the analysis data that can be found on the SEC website is the result of analyses of standards documents conducted by a minimum of three analysts [11]. Therefore my results will not match precisely with those published by the WCER.

It is also important to note that in the SCMF, many of the standards are simply listed as mathematics topics and lack specific examples of student tasks. In order to get a better sense of the cognitive demand required by each of the standards in the SCMF, I examined assessment items included in *New Elementary Mathematics Syllabus D, Book 4B*. *New Elementary Mathematics* is a series of six course textbooks written specifically for Singaporean students preparing for the GCE O-Level Exam [14]. I will include several sample assessment items from this text to justify the SEC codes I have assigned to various SCMF standards.

Similarly, to justify the coding of the CCSSM I have also included sample assessment items from Illustrative Mathematics, an initiative of the Institute for Mathematics and Education, funded by the Bill and Melinda Gates Foundation. According to William McCallum, President of the Initiative and CCSSM author, Illustrative Mathematics is “a discerning community of educators dedicated to the coherent learning of mathematics” [12]. The Illustrative Mathematics website contains resources and student assessment items that have been carefully reviewed to ensure alignment with the CCSSM.

CHAPTER 4: COMPARING THE STANDARDS

4.1 Pentagon Framework Versus Strands of Mathematical Proficiency

In many respects the pentagonal structure of the SMCF and its inter-related components resemble the strands of mathematical proficiency given in *Adding It Up*. *Mathematics Education: The Singapore Journey* notes that this resemblance, "shows parallel thinking between mathematics educators in Singapore and the U.S." [6]. From a structural viewpoint, both models consist of five inter-connected components which are highly dependent on one another and without any single component the structure of both the pentagon and the rope begin to deteriorate.

Both the strands and the pentagon framework include knowledge, skills, abilities, and beliefs that all students should acquire to be successful in mathematics education. In fact we can make the following comparisons between the strands of mathematical proficiency and the components of the pentagon framework:

- *conceptual understanding* versus *concepts;*
- *procedural fluency* versus *skills;*
- *adaptive reasoning* versus *processes;*
- *productive disposition* versus *attitudes;* and
- *strategic competence* versus *metacognition.*

The first strand *conceptual understanding* is defined as the "comprehension of mathematical concepts, operations, and relations" [22]. The authors of the strands believe that students with conceptual understanding have the ability to organize their mathematical knowledge in a way that allows them to make connections between new mathematical ideas and prior knowledge. Conceptual understanding is essential because it helps students avoid errors in mathematical thought. For example, if a student were to multiply the number 9.73 by 6.89 and arrive at the product 6703.97 the student's conceptual understanding should lead to the realization that the answer is incorrect since multiplying two numbers less than 10 and 7 respectively should result in a product less than 70. The *concepts* component of the pentagon framework shares these ideas in that it is the goal for students "to develop a deep understanding of mathematical concepts and to make sense of various mathematical ideas as well as their connections and applications" [18].

The second strand, *procedural fluency*, refers to the "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" [22]. The *skills* component of the pentagon framework refers to students' ability to carry out "numerical calculation, algebraic manipulation, spatial visualization, data analysis, measurement, use of mathematical tools, and estimation" when necessary and appropriate. One important difference is that the Singapore framework notes that, "skills should be taught with an understanding of the underlying mathematical principles and not merely as procedures" [18]. Although this may be implicit in the strands, the pentagon framework specifically makes note of this distinction. If students are taught procedural methods for solving problems without proper understanding of the mathematical concepts involved, as topics increase in difficulty it becomes harder and harder for students to understand how new material relates to old if a proper foundational understanding is absent.

Adaptive reasoning, the third strand, refers to a student's "capacity for logical thought, reflection, explanation, and justification" [22]. The component *processes* from the pentagon framework is defined as "reasoning, communication and connections, application and modeling, and thinking skills and heuristics" [18]. Reasoning is further defined as the "ability to analyze mathematical situations and construct logical arguments" [18] which closely resembles the goal of the corresponding strand. Here an important difference to note is that the pentagon framework includes a subsection within this component dedicated to mathematical modeling. It provides the flow chart, as seen below in Figure 6 [18], regarding the mathematical modelling process and further emphasizes the importance of challenging students to make connections between real-world problems and mathematical models that can represent such problems.

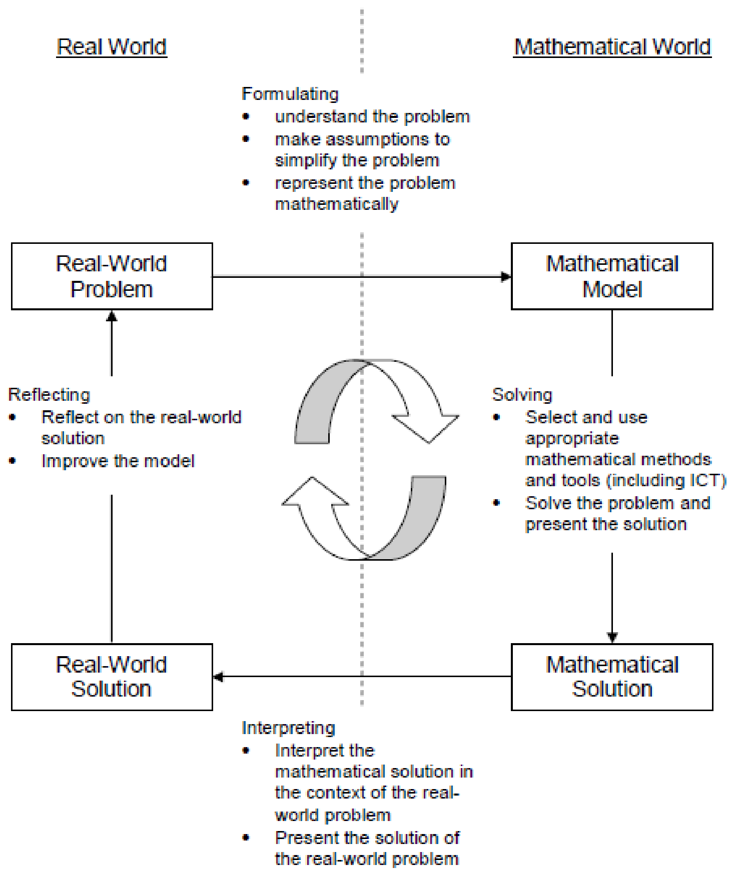


Figure 6: Mathematical Modelling Process from Pentagon Framework [18]

Productive disposition, the fourth strand, corresponds to the *attitudes* component of the pentagon framework. *Productive dispositions* is defined as the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" [22] which highly resembles *attitudes* being defined as "beliefs about mathematics and its usefulness" [18]. Both the strands and the pentagon framework stress that students' attitudes and dispositions towards mathematics are molded by the learning experiences in which they encounter. From an early age it is important that students be given with learning activities that are challenging and require perseverance and at the same time build students' confidence and help them to develop positive attitudes towards mathematics.

The fifth strand, *strategic competence*, has topics in common with the framework component *metacognition. Strategic competence* refers to a student's "ability to formulate, represent, and solve mathematical problems" [22], and *metacognition* refers to "the awareness of and ability to control one's thinking processes, in particular the selection and use of problem-solving strategies" [18]. The authors of both standards documents further explain that students who have *strategic competence* and *metacognition* should know various methods for solving a given problem as well as the ability to select the method that will be most appropriate for the given problem and conditions. This is referred to as "flexibility of approach" by the authors of the strands [22]. In addition, both the strands and the pentagon framework stress the importance of giving students the opportunity to solve non-routine problems, or problems for which students may not immediately recognize a strategy. Students with strategic competence and metacognition would be able to develop several methods for solving non-routine problems and would be able to choose flexibly between the methods depending on the conditions given in the problem.

In summary, the strands of mathematical proficiency and the pentagonal structure of the SMCF both incorporate skills and abilities that are to be developed in order for students to be successful in mathematics. Both models demonstrate an interconnectedness between components and emphasize that for students to learn mathematics effectively they must exhibit an understanding all of aspects of their respective model.

4.2 Mathematical Content Standards

Presented in Table 1 and Figure 7 below are the results based on the calculation of the total number of content standards per domain for grades seven and eight.

Domain	**CCSSM Number of Standards**	**SMCF Number of Standards**
Algebra	10 (41.7%)	41 (68.3%)
Geometry	6 (25%)	16 (26.7%)
Statistics and Probability	8 (33.3%)	3 (5%)
Total	**24**	**60**

Table 1: Number of Overall Content Standards Per Domain (Grade 7 Equivalent)

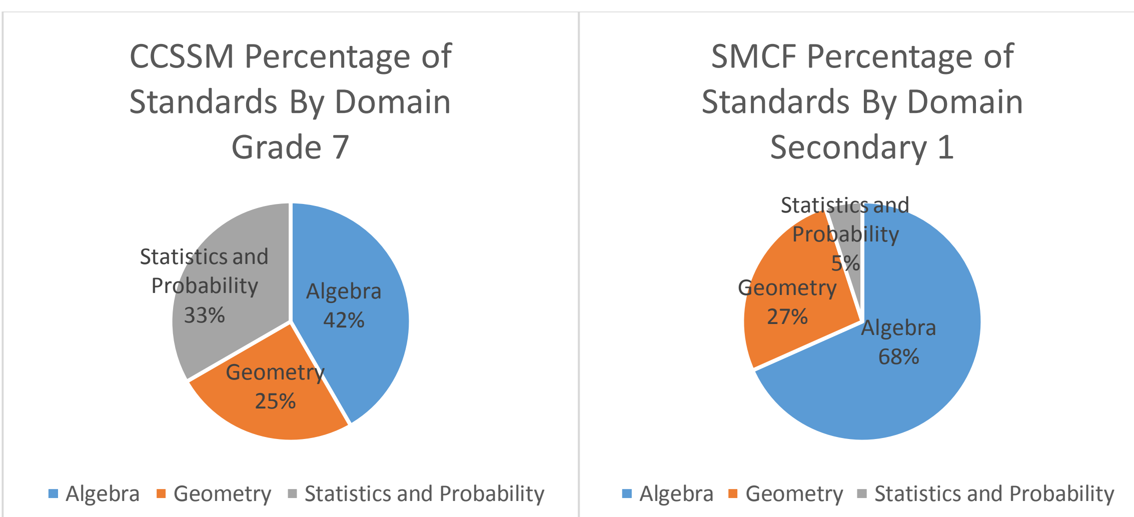


Figure 7: Percentage of Standards by Domain (Grade 7 Equivalent)

Based only on counting the number of standards, there are several differences that can be noticed between the CCSSM and SMCF. In grade seven the CCSSM has a greater focus on statistics and probability whereas the SMCF does not place as much emphasis on standards in this domain. The percentage of standards that focus on geometry topics is nearly identical for both standards documents. The percentage of standards that focus on algebra topics differs between the two documents with the SMCF having a greater percentage of standards devoted to this domain.

The results for grade eight and secondary two are represented in the Table 2 and Figure 8 below.

Domain	**CCSSM Number of Standards**	**SMCF Number of Standards**
Algebra	15 (53.6%)	20 (48.9%)
Geometry	9 (32.1%)	13 (31.6%)
Statistics and Probability	4 (14.3%)	8 (19.5%)
Total	**28**	**41**

Table 2: Number of Overall Content Standards Per Domain (Grade 8 Equivalent)

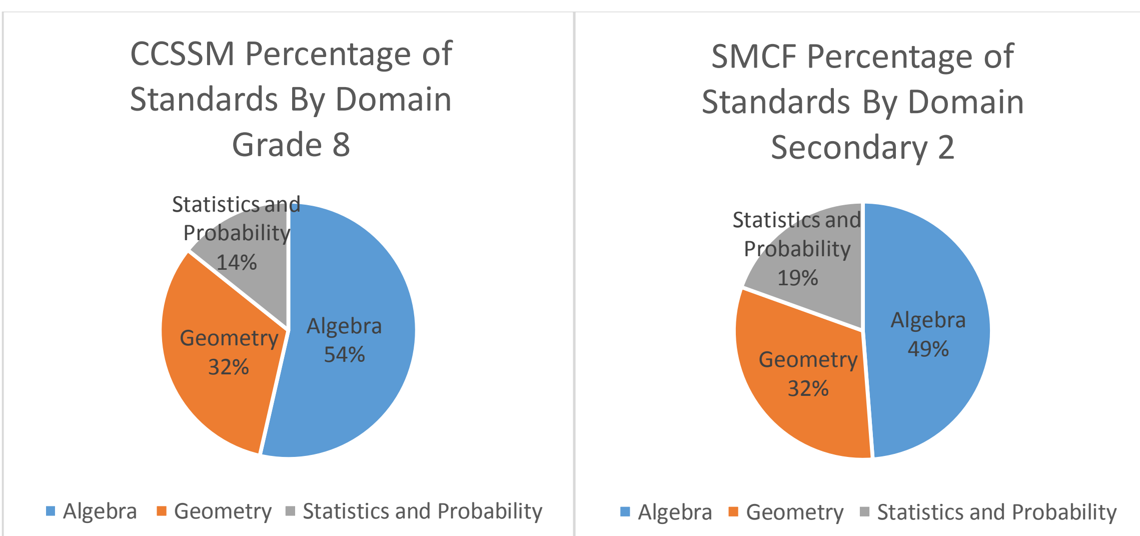


Figure 8: Percentage of Standards by Domain (Grade 8 Equivalent)

Results for grade eight and secondary two are very similar. The percentage of standards devoted to geometry, similar to grade seven and secondary one, is essentially the same. The difference in percentage of standards devoted to algebra and probability and statistics is significantly less than in grade seven.

Simply counting the number of standards that correspond to the mathematics topics, calculating their respective percentages, and making comparisons between the CCSSM and SMCF is not enough to assess the alignment of the two standards documents. So to further assess the alignment I have coded the standards documents using the SEC analysis method. For each grade level the full list of SEC codes that I assigned to each standard is listed in Appendices G-J. For increased readability of the SEC coding tables, similar to the conventions of the SEC website, I have also color-coded data with the following colors and percentages representing the overall percentage of mathematics instructional time:

= Not covered

= < 2.5%

= < 5.0%

= < 7.5%

= ≥ 7.5%

4.2.1 CCSSM Grade 7 (SMCF Secondary 1)

The SEC analysis data for CCSSM grade 7 and SMCF secondary one is displayed below in Table 3 and Table 4 respectively.

n* = 70** ***Topics	***Categories of Cognitive Demand***					***Totals***
	Memorize	Perform Procedures	Demonstrate Understanding	Conjecture, Analyze	Solve Non-Routine Problems	
Number sense/Properties/ Relationships	0%	2.9%	2.9%	0%	1.4%	**7.2%**
Operations	0%	11.4%	11.4%	1.4%	0%	**24.2%**
Measurements	1.4%	8.6%	0%	0%	2.9%	**12.9%**
Consumer Applications	0%	4.3%	0%	0%	0%	**4.3%**
Basic Algebra	0%	5.7%	4.3%	0%	5.7%	**15.7%**
Advanced Algebra	0%	0%	0%	0%	0%	**0%**
Geometric Concepts	1.4%	4.3%	2.9%	0%	5.7%	**14.3%**
Advanced Geometry	0%	0%	0%	1.4%	0%	**1.4%**
Data Displays	0%	2.9%	0%	0%	0%	**2.9%**
Statistics	0%	0%	0%	4.3%	0%	**4.3%**
Probability	0%	1.4%	5.7%	4.3%	0%	**11.4%**
Analysis	0%	0%	0%	0%	0%	**0%**
Trigonometry	0%	0%	0%	0%	0%	**0%**
Special Topics	0%	0%	0%	0%	0%	**0%**
Functions	0%	0%	0%	0%	0%	**0%**
Instructional Technology	0%	1.4%	0%	0%	0%	**1.4%**
Totals	**2.8%**	**42.9%**	**27.2%**	**11.4%**	**15.7%**	**100%**

Table 3: CCSSM Grade 7

n* = 122** ***Topics	***Categories of Cognitive Demand***					***Totals***
	Memorize	Perform Procedures	Demonstrate Understanding	Conjecture, Analyze	Solve Non-Routine Problems	
Number sense/Properties/ Relationships	1.6%	13.1%	1.6%	0%	0%	**16.3%**
Operations	0%	5.7%	0.8%	0%	0%	**6.5%**
Measurements	0.8%	10.7%	0%	0.8%	1.6%	**13.9%**
Consumer Applications	0%	0%	0%	0%	0.8%	**0.8%**
Basic Algebra	0%	13.9%	6.6%	2.5%	4.9%	**27.9%**
Advanced Algebra	0%	0%	0%	0%	0%	**0%**
Geometric Concepts	12.3%	6.6%	0.8%	0%	2.5%	**22.2%**
Advanced Geometry	0%	0.8%	0%	0%	0%	**0.8%**
Data Displays	0%	0%	0%	6.6%	0%	**6.6%**
Statistics	0.8%	0%	0%	1.6%	0%	**2.4%**
Probability	0%	0%	0%	0%	0%	**0%**
Analysis	0%	0%	0%	0%	0%	**0%**
Trigonometry	0%	0%	0%	0%	0%	**0%**
Special Topics	0%	0%	0%	0%	0%	**0%**
Functions	0%	1.6%	0%	0%	0%	**1.6%**
Instructional Technology	0%	0.8%	0%	0%	0%	**0.8%**
Totals	**15.5%**	**53.2%**	**9.8%**	**11.5%**	**9.8%**	**99.8%**

Table 4: SMCF Secondary 1

For CCSSM, based on the results of the SEC coding for grade seven and secondary one, we notice several differences in terms of topic and cognitive demand. In terms of topic, the SMCF focuses more on the topics of *basic algebra* (27.9%) and *geometric concepts* (22.2%), while the CCSSM focuses more on the topic of *operation* (24.2%) and slightly less on both *basic algebra* (15.7%) and *geometric concepts* (14.3%). Another significant difference is that CCSSM has a greater focus on *probability* (11.4%) whereas the standards in the SMCF show no alignment with this topic.

In terms of cognitive demand, the SEC coding revealed that a greater percentage of standards in the SMCF require lower levels of cognitive demand. For the cognitive demand category, *memorize*, it was found that only 2.8% of the standards in the CCSSM require this level of cognitive demand while 15.5% of standards in the SMCF require this level of cognitive demand. The cognitive demand category, *perform procedures*, shows similar results with 42.9% of CCSSM standards and 53.2% of SMCF in this category. Higher cognitive demand levels including *demonstrate understanding* (27.2%) and *solve non-routine problems* (15.7%) account for larger percentages of standards in the CCSSM as compared with the SMCF (9.8% and 9.8%, respectively). The percentage of standards requiring the cognitive demand level *conjecture/analyze* is nearly identical (11.4% and 11.5%, respectively) for both standards documents.

To support my SEC coding for grade seven and secondary one I have included sample assessment items for each standards document.

For SCMF standard G.1.6, "angle sum of interior and exterior angles of any convex polygon" [18], I coded this standard 710B, 711B, 710C, and 711C. 710 and 711 refer to the mathematics subtopics *angles* and *polygons,* respectively, which are both under the broad mathematics topic *geometric concepts*. The letters B and C refer to the cognitive demand categories *memorize* and *perform procedures,* respectively. The assessment item in Figure 9 appears to expect that students recall the sum of the interior angles of the various regular polygons and then use this knowledge to calculate the base angle of the isosceles triangle. Instead of recalling the sum of the interior angles of each regular polygon students could derive it, but we note that the assessment item does not explicitly require this (although it does forbid it either).

The figure is made up of a regular pentagon, a regular hexagon and an isosceles triangle. Calculate the base angle of the triangle.

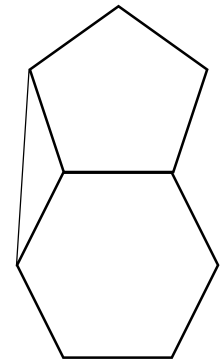
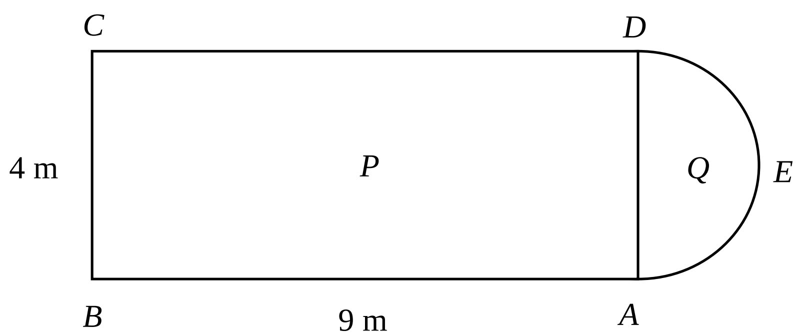


Figure 9: Assessment Item 1 from *New Elementary Mathematics Syllabus D, Book 4B* (p. 63) [14]

For SCMF standard G.5.2, "problems involving perimeter and area of plane figures" [18], I assigned the following SEC codes: 305C, 306C, and 790C, based on the assessment item in Figure 10. In this problem students are required to perform procedures and follow instructions to find various measurements, including area and perimeter, of the given geometric figure.



A flower bed *ABCDE*, shown in the diagram, consists of two parts *P* and *Q*. The part *P* is rectangular and measures 9 m by 4 m and the part *Q* is semicircular.

(a) Write down the radius of the semicircle.

(b) Taking π to be 3, find the length of the arc of the semicircle *AED*.

(c) Find the perimeter of the flower bed.

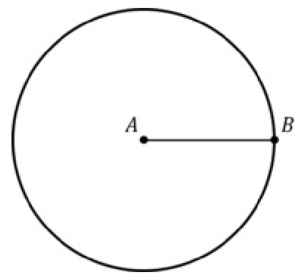
(d) Find the area of *P*.

(e) Taking π to be 3, calculate the area of the whole flower bed.

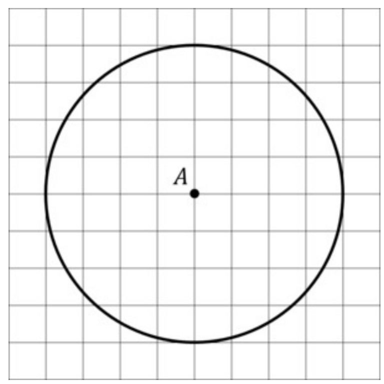
Figure 10: Assessment Item 2 from *New Elementary Mathematics Syllabus D, Book 4B* (p. 73) [14]

For the CCSSM standard 7.G.B.4, students are required to "know the formulas for area and circumference of a circle and use them to solve problems; give an informal derivation of the relationships between the circumference and area of a circle" [20]. I assigned the SEC codes 310B and 801E to this standard. 310 refers to subtopic *circles (e.g., pi, radius, area)* under the broad mathematics topic *measurement* and 801 refers to the subtopic *logic, reasoning, and proofs* under the broad mathematics topic *advanced geometry.* The letters B and E refer to the cognitive demand levels *memorize* and *conjecture/analyze*. Figure 11 below is a sample assessment item from Illustrative Mathematics which aligns with CCSSM standard 7.G.B.4. This assessment item is designed to help students "differentiate between a circle and the region inside of the circle so that they understand what is being measured when the circumference and area are being found" [12]. This assessment item allows students to actively participate and estimate the circumference and area of the given circle and then investigate their conjectures rather than simply calculate results given the appropriate formulas.

1. What is the definition of a circle with center *A* and radius *r*?
2. A circle has center *A* and radius *AB*. Is point *A* on the circle? Is point *B* on the circle? Explain.



3. Imagine that a circle with center *A* is drawn on 1/4 inch grid paper as shown below. What is the radius of the circle?



4. Use the grid to estimate the circumference of the circle.
5. Use the grid to estimate the area of the region enclosed by the circle.
6. What are you measuring when you find the circumference of a circle? What are you measuring when you find the area of a circle?

Figure 11: "The Circumference of a Circle and the Area of the Region it Encloses" Assessment Item from Illustrative Mathematics [12]

In general the CCSSM and the SMCF content for grade seven and secondary one is similar in topic, but differs in cognitive demand. It can be seen from the SEC coding and from the assessment items presented that the CCSSM more frequently requires students to make connections between mathematical concepts which necessitates a deeper level of understanding of mathematical ideas.

4.2.2 CCSSM Grade 8 (SMCF Secondary 2)

The SEC analysis data for CCSSM grade 8 and SMCF secondary two is displayed below in Table 5 and Table 6 respectively.

n* = 64** ***Topics	***Categories of Cognitive Demand*** Memorize	Perform Procedures	Demonstrate Understanding	Conjecture, Analyze	Solve Non-Routine Problems	***Totals***
Number sense/Properties/ Relationships	4.7%	15.6%	0%	0%	0%	**20.3%**
Operations	0%	1.6%	0%	0%	0%	**1.6%**
Measurements	1.6%	0%	0%	0%	0%	**1.6%**
Consumer Applications	0%	0%	0%	0%	0%	**0%**
Basic Algebra	0%	14.1%	7.8%	0%	0%	**21.9%**
Advanced Algebra	0%	0%	0%	1.6%	1.6%	**3.2%**
Geometric Concepts	0%	4.7%	7.8%	6.3%	1.6%	**20.4%**
Advanced Geometry	1.6%	0%	0%	0%	1.6%	**3.2%**
Data Displays	1.6%	1.6%	0%	1.6%	0%	**4.8%**
Statistics	0%	3.1%	1.6%	3.1%	0%	**7.8%**
Probability	0%	0%	0%	0%	0%	**0%**
Analysis	0%	0%	0%	0%	0%	**0%**
Trigonometry	0%	0%	0%	0%	0%	**0%**
Special Topics	0%	0%	0%	0%	0%	**0%**
Functions	1.6%	1.6%	4.7%	6.3%	0%	**14.2%**
Instructional Technology	0%	0%	1.6%	0%	0%	**1.6%**
Totals	**11.1%**	**42.3%**	**23.5%**	**18.9%**	**4.8%**	**100.6%**

Table 5: CCSSM Grade 8

n = 75 *Topics*	***Categories of Cognitive Demand***					***Totals***
	Memorize	Perform Procedures	Demonstrate Understanding	Conjecture, Analyze	Solve Non-Routine Problems	
Number sense/Properties/ Relationships	0%	0%	0%	0%	0%	**1.3%**
Operations	0%	6.7%	0%	0%	0%	**6.7%**
Measurements	0%	5.3%	0%	1.3%	2.7%	**9.3%**
Consumer Applications	0%	0%	0%	0%	1.3%	**1.3%**
Basic Algebra	0%	22.7%	0%	2.7%	4.0%	**29.4%**
Advanced Algebra	0%	5.3%	0%	0%	0%	**5.3%**
Geometric Concepts	6.7%	8%	4.0%	0%	4.0%	**22.7%**
Advanced Geometry	0%	1.3%	0%	0%	0%	**1.3%**
Data Displays	0%	0%	0%	9.3%	0%	**9.3%**
Statistics	1.3%	2.7%	0%	2.7%	0%	**6.7%**
Probability	0%	1.3%	0%	1.3%	0%	**2.6%**
Analysis	0%	0%	0%	0%	0%	**0%**
Trigonometry	0%	1.3%	0%	0%	0%	**1.3%**
Special Topics	0%	0%	0%	0%	0%	**0%**
Functions	0%	2.7%	0%	0%	0%	**2.7%**
Instructional Technology	0%	0%	0%	0%	0%	**0%**
Totals	**8.0%**	**58.6%**	**4.0%**	**17.3%**	**12.0%**	**99.9%**

Table 6: SMCF Secondary 2

Based on the results of the SEC coding for grade eight and secondary two, we again notice differences in terms of topic and cognitive demand. First, in terms of topic, for both the CCSSM and SMCF the main topics of focus are *basic algebra* and *geometric concepts*, but the CCSSM also has a large focus on *number sense* (20.3%) and *functions* (14.2%) while the respective percentages for the SMCF are far less (1.3% and 2.7%, respectively).

In terms of cognitive demand, some of the observations from the analysis of grade seven and secondary one are still true here. For both the CCSSM and the SMCF the cognitive demand category *perform procedures* comprises the largest percentage of standards (42.3% and 58.6%, respectively), but the SMCF still has a larger focus on this cognitive demand than the CCSSM. One significant difference in the standards at this level is that the CCSSM requires students to demonstrate a deeper level of understanding of mathematical ideas than does the SMCF. For example, the CCSSM standard 8.G.2, requires that students [20]

> Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Similarly, the CCSSM standard 8.G.4., requires that students [20]

> Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

One the other hand, the SMCF secondary two standard G.2.6 only requires that students are capable of "solving simple problems involving congruence and similarity" [18]. Similarly the CCSSM standard 8.G.B.6 requires that students, "explain a proof of the Pythagorean Theorem and its converse" [20]. The SMCF secondary two standard G.4.1 only requires the "use of Pythagoras' theorem" [18].

As previously mentioned, the SEC has a limited number of published content analysis results available on their official website. For CCSSM grade eight and SMCF secondary one, such results are available. Appendix I [26] includes a comparison table displaying these analysis results. In most respects, the results of my analysis align with those published by the SEC; perfect alignment would be difficult since the published results are the average of the coding of a minimum of three analysts.

For secondary two, I will also include a sampling of assessment items for which I based SEC coding.

For SMCF secondary two standard N.7.7, “solving simultaneous linear equations in two variables” [18], I assigned the SEC code 602C. 602 refers to the mathematics subtopic *systems of equations* under the broad mathematics topic *advanced algebra*, and the letter C refers to the cognitive demand *perform procedures*. I based this code assignment on the assessment item in Figure 12. This assessment items requires students to perform procedures to solve a routine system of linear equations.

Ann and Betty went marketing together.

(a) Ann bought 400 g of prawns and 1 kg 300 g of fish for $17.90. This information can be expressed as:

$$0.4x + 1.3y = 17.9$$

What do the letters *x* and *y* stand for?

(b) Betty bought twice as much of the same type of prawns and half as much of the same type of fish for $19.03. Write down an equation to represent this information.

(c) Use the two equations in parts (a) and (b) to find the price per kg of

(i) the prawns,

(ii) the fish.

Figure 12: Assessment Item 3 from *New Elementary Mathematics Syllabus D, Book 4B* (p. 36) [14]

The CCSSM contains an analogous standard 8.HEE.C.8 [20],

Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
c. Solve real-world mathematical problems leading to two linear equations in two variables.

I assigned the SEC codes 505C, 507C, 602E, and 690F to this standard. 505 and 507 refer to the subtopics *coordinate planes* and *multi-step equations* under the broad mathematics topic *basic algebra*, and 602 refers to the subtopic *systems of equations* and 690 refers to *other* under the broad topics *advanced algebra*. The letters C, E, and F refer to the cognitive demand categories *perform procedures, conjecture/analyze,* and *solve non-routine problems.* This coding was based on the sample assessment item in Figure 13. This assessment item requires that students analyze the information given in order to develop a system of equations in two variables to solve the non-routine problem. This assessment item requires a higher cognitive demand level in comparison to the assessment item in Figure 12 in that students are required to develop both equations that comprise the system; the assessment item in Figure 12 establishes one equation for students for which they can draw from to develop the second equation. In addition, the assessment item in Figure 13 requires students to reason about their solution and suggests that students consider multiple ways to approach the problem both of which will lead to a deeper understanding of the underlying concepts.

Ivan's furnace has quit working during the coldest part of the year, and he is eager to get it fixed. He decides to call some mechanics and furnace specialists to see what it might cost him to have the furnace fixed. Since he is unsure of the parts he needs, he decides to compare the costs based only on service fees and labor costs. Shown below are the price estimates for labor that were given to him by three different companies. Each company has also given him an estimate of the time it will take to fix the furnace.

- Company A charges $35 per hour to its customers.
- Company B charges a $20 service fee for coming out to the house and then $25 per hour for each additional hour.
- Company C charges a $45 service fee for coming out to the house and then $20 per hour for each additional hour.

For which time intervals should Ivan choose Company A, Company B, Company C? Support your decision with sound reasoning and representations. Consider including equations, tables, and graphs.

Figure 13: "Fixing the Furnace" Assessment Item from Illustrative Mathematics [12]

For SMCF standards G.4.1, "use of Pythagoras' theorem" [18], and G.5.6, "volume and surface area of pyramid, cone, and sphere" [18], I assigned the SEC codes 717D and 306C, 307C, 712C, and 803C, respectively. I chose these SEC codes based on the assessment item in Figure 14. This assessment item requires that students perform procedures using mathematical concepts of slant height, volume, and surface area of a cone, in addition to demonstrating an understanding between slant height and the Pythagorean Theorem.

In this question, take π to be 3.14.

In the diagram, the vertical height of the cone is 12 cm and the diameter of its base is 10 cm. Calculate

(a) the slant height of the cone,

(b) the volume of the cone,

(c) the total surface area of the cone.

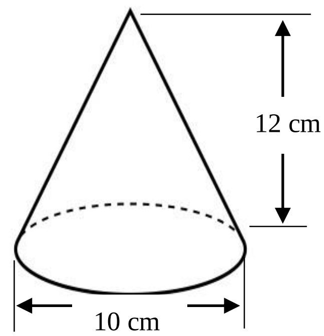
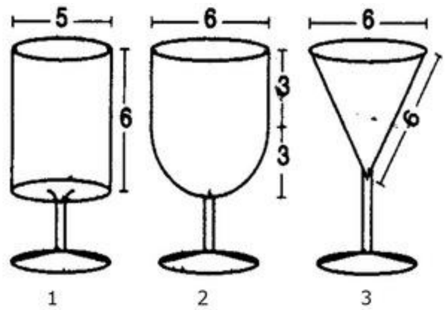


Figure 14: Assessment Item 4 from *New Elementary Mathematics Syllabus D, Book 4B* (p. 70) [14]

The CCSSM grade 8 standards G.B.7 and G.C.9 require that students be able to "apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions" [20] and that students "know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems" [20]. To standard G.B.7 I assigned SEC codes 717C and 717F, and to standard G.C.9 I assigned SEC codes 306B, 803B, and 803F based on the Illustrative Mathematics assessment item in Figure 15. The assessment item in Figure 15 requires that students know and use the volume formula for cones, cylinders, and spheres. Also, the assessment item requires students to solve a non-routine problem that increases in difficulty as students progress through the problem from one glass to another; students start with simply performing a procedure to calculate the volume of the cylindrical glass and move to more difficult tasks that require multiple formulas and the use of the Pythagorean Theorem.

The diagram shows three glasses (not drawn to scale). The measurements are all in centimeters.



The bowl of glass 1 is cylindrical. The inside diameter is 5 cm and the inside height is 6 cm.

The bowl of glass 2 is composed of a hemisphere attached to cylinder. The inside diameter of both the hemisphere and the cylinder is 6 cm. The height of the cylinder is 3 cm.

The bowl of glass 3 is an inverted cone. The inside diameter is 6 cm and the inside slant height is 6 cm.

a. Find the vertical height of the bowl of glass 3.
b. Calculate the volume of the bowl of each of these glasses.
c. Glass 2 is filled with water and then half the water is poured out. Find the height of the water.

Figure 15: “Glasses” Assessment Item from Illustrative Mathematics [12]

For the SMCF standards S.1.7, “mean, mode, and median as measures of central tendancy for a set of data” [18], I assigned the SEC code 1001C. 1001 refers to the mathematics subtopic *mean, median, and mode* under the broad mathematics topic *statistics*, and the letter C refers to *performing procedures.* I chose to code this standard based on the assessment item in Figure 16. This assessment item requires that students perform procedures in a routine word problem based on the data in the table to find the mode, median, and mean of the given data.

A man throws 2 dice and records the total score. The results of 50 throws are shown in the following table.

Score	2	3	4	5	6	7	8	9	10	11	12
No. of times	1	1	2	5	6	8	7	6	6	5	3

Find **(a)** the mode,

(b) the median,

(c) the mean score.

Figure 16: Assessment Item 5 from *New Elementary Mathematics Syllabus D, Book 4B* (p. 185) [14]

In general, content analysis results for CCSSM grade eight and SMCF secondary two are similar to those of grade seven and secondary one. Again SEC coding and sample assessment items demonstrate that the CCSSM continue to require a slightly higher level of cognitive demand than the SMCF.

4.2.3 CCSSM Grades 9-12 (Singapore O-Level Exam Syllabus): Number and Algebra Conceptual Category/Content Strand

The SEC analysis data for CCSSM grades nine through twelve and Singapore secondary one through four for the mathematics number and algebra conceptual category/content strand is displayed below in Table 7 and Table 8 respectively.

n* = 62** ***Topics	***Categories of Cognitive Demand*** Memorize	Perform Procedures	Demonstrate Understanding	Conjecture, Analyze	Solve Non-Routine Problems	***Totals***
Number sense/Properties/ Relationships	0%	1.6%	8.1%	0%	0%	**9.7%**
Operations	0%	1.6%	1.6%	0%	0%	**3.2%**
Measurements	0%	0%	0%	0%	0%	**0%**
Consumer Applications	0%	0%	0%	0%	0%	**0%**
Basic Algebra	0%	32.2%	9.7%	3.2%	0%	**45.1%**
Advanced Algebra	1.6%	17.8%	0%	4.8%	0%	**24.2%**
Geometric Concepts	0%	0%	0%	0%	0%	**0%**
Advanced Geometry	0%	0%	0%	0%	0%	**0%**
Data Displays	0%	0%	0%	0%	0%	**0%**
Statistics	0%	0%	0%	0%	0%	**0%**
Probability	0%	0%	0%	0%	0%	**0%**
Analysis	0%	0%	0%	1.6%	0%	**1.6%**
Trigonometry	0%	0%	0%	0%	0%	**0%**
Special Topics	0%	0%	0%	0%	0%	**0%**
Functions	0%	4.8%	8.1%	0%	0%	**12.9%**
Instructional Technology	0%	3.2%	0%	0%	0%	**3.2%**
Totals	**1.6%**	**61.7%**	**30.7%**	**9.6%**	**0%**	**99.9%**

Table 7: CCSSM Grades 9-12 Number and Algebra Conceptual Category

n = 117 ***Topics***	***Categories of Cognitive Demand***					***Totals***
	Memorize	Perform Procedures	Demonstrate Understanding	Conjecture, Analyze	Solve Non-Routine Problems	
Number sense/Properties/ Relationships	2.6%	17.1%	0.9%	0%	0%	**20.6%**
Operations	0%	10.2%	0.9%	0%	0%	**11.1%**
Measurements	1.7%	6.0%	0%	0%	0.9%	**8.6%**
Consumer Applications	0%	0%	0%	0%	1.7%	**1.7%**
Basic Algebra	0%	24.0%	5.1%	0.9%	4.3%	**34.3%**
Advanced Algebra	0.9%	7.7%	0.9%	0%	0%	**9.5%**
Geometric Concepts	0%	0%	0%	0%	0%	**0%**
Advanced Geometry	0%	0%	0%	0%	0%	**0%**
Data Displays	0%	3.4%	0%	0%	0.9%	**4.3%**
Statistics	1.7%	0.9%	0%	0%	0%	**2.6%**
Probability	0%	0%	0%	0%	0%	**0%**
Analysis	0%	0%	0%	0%	0%	**0%**
Trigonometry	0%	0%	0%	0%	0%	**0%**
Special Topics	1.7%	0%	0%	0%	0%	**1.7%**
Functions	0%	5.1%	0%	0%	0%	**5.1%**
Instructional Technology	0%	0.9%	0%	0%	0%	**0.9%**
Totals	**8.6%**	**75.3%**	**7.8%**	**0.9%**	**7.8%**	**100.4%**

Table 8: SMCF O-Level Exam Syllabus Number and Algebra Content Strand

The SEC content analysis results for both standards documents in the number and algebra conceptual category/content strand for secondary grades, shows differences in terms of mathematics topics. One similarity however, is that the highest percentage of standards for both documents is on *basic algebra* (45.1% and 34.3% for CCSSM and SMCF, respectively). The CCSSM then focuses on the topics *advanced algebra* (24.2%) and *functions* (12.9%). The SMCF focuses instead on the topics *number sense* (20.6%) and *operations* (11.1%). This difference does not mean that the analogous standards are absent from the CCSSM. In fact, most of the SMCF standards that focus on the topics *number sense* and *operations* can be found in the CCSSM in earlier grades. Another significant difference is that the SMCF includes standards related to matrices which are to be included in the curriculum for all O-Level students whereas the CCSSM includes analogous standards but indicates that the standards are additional mathematics and are not required of all students.

In terms of cognitive demand, for both standards documents, the highest percentage of standards correspond to the cognitive demand category *perform procedures* (61.7% and 75.3% for CCSSM and SMCF, respectively). The greatest differences in the cognitive demand levels of the standards in the number and algebra conceptual category/content strand occurs in the cognitive demand categories *demonstrate understanding* (30.7% and 7.8% for CCSSM and SMCF, respectively) and *conjecture/analyze* (9.6% and 0.9% for CCSSM and SMCF, respectively).

For the CCSSM standard HSA.REI.C.6, "solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables" [20], the SEC code 602C was assigned. 602 refers to the subtopic *systems of equations* within the topic *advanced algebra*, and the letter C refers to the cognitive demand category *perform procedures*. The Illustrative Mathematics assessment item in Figure 17 below, was found to align with standard HSA.REI.C.6 [12]. This assessment item requires that students develop a system of two equations in two unknowns and use that system to solve the given problem. This assessment item is largely procedural, but Illustrative Mathematics suggests a variety of ways to increase the cognitive demand level including a physical simulation of the problem and/or a small group discussion of alternate ways to approach and solve the problem [12].

Nola was selling tickets at the high school dance. At the end of the evening, she picked up the cash box and noticed a dollar lying on the floor next to it. She said,

I wonder whether the dollar belongs inside the cash box or not.

The price of tickets for the dance was 1 ticket for \$5 (for individuals) or 2 tickets for \$8 (for couples). She looked inside the cash box and found \$200 and ticket stubs for the 47 students in attendance. Does the dollar belong inside the cash box or not?

Figure 17: "Cash Box" Assessment Item from Illustrative Mathematics [12]

The SMCF standard N.1.8 includes "solving simultaneous linear equations in two unknowns by substitution and elimination methods and graphical method" [23]. This standard was also assigned the SEC code 602C. Figure 18 below includes a sample assessment item which aligns with this standard. Similar to the Illustrative Mathematics assessment item, this item requires that students create a system of linear equations in two unknowns and then use that system to provide solutions to the given questions.

Alice bought 120 plums at x cents each and 100 peaches at y cents each. She put 6 plums and 5 peaches in each bag and sold the bags for $(9x + 6y)$ cents each.

(a) Write down, in terms of x and y, an expression for
  i) The amount of money, in dollars, she spent on fruit,
  ii) The total amount of money, in dollars, she received from selling her bags of fruit.
(b) Given that her cost was \$80 and she made a profit of 38%, find the value of x and y.

Figure 18: Assessment Item 6 from *New Elementary Mathematics Syllabus D, Book 4B* (p. 29) [14]

For the CCSSM standard HSA.CED.A.4, "rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations" [20], and the SMCF standard 1.6 "changing the subject of a formula" [23], SEC codes 507C and 516C were assigned to both standards. 507 and 516 refer to subtopics *multi-step equations* and *multiples representations* within the topic *basic algebra* with the letter C corresponding to the cognitive domain category *perform procedures.* Figures 19 and 20 below present sample assessment items aligning to these standards. Both assessment items are procedural in design and require using the four operations to solve equations for an indicated variable in terms of other variables. However, the CCSSM assessment item in Figure 19 includes a progression of equations, starting with simpler equations involving fewer steps and eventually leading to more difficult equations, whereas the SMCF assessment item requires students to rearrange a single formula without any progression.

Use inverse operations to solve the equations for the unknown variable, or for the designated variable if there is more than one. If there is more than one operation to "undo", be sure to think carefully about the order in which you do them. For equations with multiple variables, it may help to first solve a version of the problem with numerical values substituted in.

1. $5 = a - 3$
2. $A - B = C$ (solve for *A*)
3. $6 = -2x$
4. $IR = V$ (solve for *R*)
5. $\frac{x}{5} = 3$
6. $W = \frac{A}{L}$ (solve for *A*)
7. $7x + 3 = 10$
8. $ax + c = R$ (solve for *x*)
9. $13 = 15 - 4x$
10. $2h = w - 3p$ (solve for *p*)
11. $F = \frac{GMm}{r^2}$ (solve for *G*)

Figure 19: "Equations and Formulas" Assessment Item from Illustrative Mathematics [12]

Make *h* the subject of the formula $d = \frac{4ftV^2}{2gh - aV^2}$

Figure 20: Assessment Item 7 from *New Elementary Mathematics Syllabus D, Book 4B* (p. 26) [14]

4.2.4 CCSSM Grades 9-12 (Singapore O-Level Exam Syllabus): Geometry Conceptual Category/Content Strand

The SEC analysis data for CCSSM grades nine through twelve and Singapore secondary one through four for the mathematics conceptual category/content strand geometry is displayed below in Table 9 and Table 10 respectively.

***n* = 89**	***Categories of Cognitive Demand***					
Topics	Memorize	Perform Procedures	Demonstrate Understanding	Conjecture, Analyze	Solve Non-Routine Problems	***Totals***
Number sense/Properties/ Relationships	0%	0%	0%	0%	0%	**0%**
Operations	0%	0%	0%	0%	0%	**0%**
Measurements	0%	0%	0%	0%	0%	**0%**
Consumer Applications	0%	0%	0%	0%	0%	**0%**
Basic Algebra	0%	0%	0%	0%	0%	**0%**
Advanced Algebra	0%	0%	0%	0%	0%	**0%**
Geometric Concepts	2.2%	21.3%	25.8%	16.9%	2.2%	**68.4%**
Advanced Geometry	0%	3.4%	0%	14.6%	1.1%	**19.1%**
Data Displays	0%	0%	0%	0%	0%	**0%**
Statistics	0%	0%	0%	0%	0%	**0%**
Probability	0%	0%	0%	0%	0%	**0%**
Analysis	0%	0%	0%	0%	0%	**0%**
Trigonometry	1.1%	2.2%	3.4%	0%	2.2%	**8.9%**
Special Topics	0%	0%	0%	0%	0%	**0%**
Functions	0%	0%	0%	0%	0%	**0%**
Instructional Technology	0%	1.1%	2.2%	0%	0%	**3.3%**
Totals	**3.3%**	**28.0%**	**31.4%**	**31.5%**	**5.5%**	**99.7%**

Table 9: CCSSM Grades 9-12 Geometry Conceptual Category

n* = 75** ***Topics	***Categories of Cognitive Demand*** Memorize	 Perform Procedures	 Demonstrate Understanding	 Conjecture, Analyze	 Solve Non-Routine Problems	***Totals***
Number sense/Properties/ Relationships	0%	1.3%	0%	0%	0%	**1.3%**
Operations	0%	0%	0%	0%	0%	**0%**
Measurements	1.3%	14.7%	0%	0%	0%	**16.0%**
Consumer Applications	0%	0%	0%	0%	0%	**0%**
Basic Algebra	0%	0%	0%	0%	0%	**0%**
Advanced Algebra	0%	0%	0%	0%	0%	**0%**
Geometric Concepts	26.7%	25.3%	5.3%	0%	0%	**57.3%**
Advanced Geometry	6.7%	10.7%	0%	0%	0%	**17.4%**
Data Displays	0%	0%	0%	0%	0%	**0%**
Statistics	0%	0%	0%	0%	0%	**0%**
Probability	0%	0%	0%	0%	0%	**0%**
Analysis	0%	0%	0%	0%	0%	**0%**
Trigonometry	0%	8.0%	0%	0%	0%	**8.0%**
Special Topics	0%	0%	0%	0%	0%	**0%**
Functions	0%	0%	0%	0%	0%	**0%**
Instructional Technology	0%	0%	0%	0%	0%	**0%**
Totals	**34.7%**	**60.0%**	**5.3%**	**0%**	**0%**	**100%**

Table 10: SMCF O-Level Exam Syllabus Geometry Content Strand

In terms of topics, the majority of the standards, for both standards documents, were assigned codes that correspond to subtopics within the topic *geometric concepts* (68.4% and 57.3% for CCSSM and SMCF, respectively) which is to be expected for standards within the geometry domain. Additionally, for the topic *advanced geometry*, roughly the same percentage of standards (19.1% and 17.4% for CCSSM and SMCF, respectively) were assigned codes that correspond to subtopics within this topic. The percentage of standards that correspond to subtopics with the topic *trigonometry* is also very similar, 8.9% and 8.0% for CCSSM and SMCF, respectively. A noticeable difference between the standards documents is the percentage of standards that focus on the topic *measurement*. For the SMCF 16.0% of standards were assigned codes corresponding to subtopics within this topic, while no standards in the CCSSM were found to correspond to subtopics within this topic.

In terms of cognitive demand, SEC content analysis results for the CCSSM and SMCF standards in the geometry conceptual category/content strand for secondary grades shows that the SMCF focuses almost entirely on the cognitive demand categories *memorize* (34.7%) and *perform procedures* (60.0%) which is in contrast to the CCSSM where the focus is more evenly distributed between the cognitive demand categories of *perform procedures* (28.0%), *demonstrate understanding* (31.4%), *and conjecture/analyze* (31.5%). Within individual topics this trend can also be seen. More specifically, within the topics of *geometric concepts*, the SMCF standards largely focus on *memorizing* (26.7%) and *performing procedures* (25.3%). Similarly for the SMCF standards within the topic *trigonometry,* all of the standards were found to focus on *performing procedures* whereas the CCSSM standards in this topic are more evenly distributed between the cognitive demand categories *memorize* (1.1%), *perform procedures* (2.2%), *demonstrate understanding* (3.4%), and *solve non-routine problems* (2.2%).

For the CCSSM standard GMD.A.3, "use volume formulas for cylinders, pyramids, cones, and spheres to solve problems" [20], I have assigned the following SEC codes: 712C, 713C, 803C, and 803F. 712 and 713 refer to the subtopics *polyhedra* and *models*, respectively, within the topic *geometric concepts,* and 803 refers to the subtopic *spheres, cones, and cylinders* within the topic *advanced geometry.* The letters C and F refer to the cognitive demand categories *perform procedures* and *solve non-routine problems*. Figure 21 below is a sample assessment item from Illustrative Mathematics that aligns with standard GMD.A.3 [12]. This assessment item requires that students know the volume formulas for cones and cylinders, and it also involves the creation and use of a geometric model to represent a real world problem. Here students are not given a model, but they must first create their own, given the conditions of the problem, and then use this model to answer the set of questions.

> Jared is scheduled for some tests at his doctor's office tomorrow. His doctor has instructed him to drink 3 liters of water today to clear out his system before the tests. Jared forgot to bring his water bottle to work and was left in the unfortunate position of having to use the annoying paper cone cups that are provided by the water dispenser at his workplace. He measures one of these cones and finds it to have a diameter of 7 cm and a slant height (measured from the bottom vertex of the cup to any point on the opening) of 9.1 cm.
>
> Note: $1\ cm^3 = 1\ ml$
>
> How many of these cones of water must Jared drink if he typically fills the cone to within 1 cm of the top and he wants to complete his drinking during the work day?
>
> 1. Suppose that Jared drinks 25 cones of water during the day. When he gets home he measures one of his cylindrical drinking glasses and finds it to have a diameter of 7 cm and a height of 15 cm. If he typically fills his glasses to 2 cm from the top, about how many glasses of water must he drink before going to bed?

Figure 21: "Doctor's Appointment" Assessment Item from Illustrative Mathematics [12]

An assessment item that is similar in topic, which aligns with the SMCF geometry and measurement standard 2.5, "volume and surface area of cube, cuboid, prism, cylinder, pyramid, cone and sphere" [23] is presented below in Figure 22. The SEC codes that were assigned to this standard are 306C, 307C, 712C, and 803C. The codes 306 and 307 refer to the subtopics *area and volume* within the topic *measurement*, 712 refers to the subtopic *polyhedra* within the topic *geometric concepts*, and 803 refers to the subtopic *spheres, cones, and cylinders* within the topic *advanced geometry.* The letter C refers to the cognitive demand category *performing procedures.* This assessment item requires that students know volume and surface area formulas for cubes and square prisms and are able to perform procedures to calculate the indicated quantities for the given composite solid.

The diagram shows a piece of crystal. Each of the edges is 2 cm long. Calculate

(a) the volume of the crystal, correct to the nearest cubic centimetres,
(b) the total surface area of the crystal, correct to the nearest square centimetres.

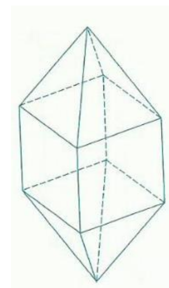
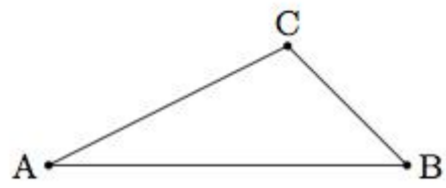


Figure 22: Assessment Item 8 from *New Elementary Mathematics Syllabus D, Book 4B* (p. 81) [14]

For CCSSM standard HSG.SRT.C.6, "understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles" [20], I assigned SEC codes 705D, 1301D, 1303D. 705 refers to the subtopic *similarity* within the topic *geometric concepts* and 1301 and 1303 refer to the subtopics *basic ratios and right triangle trigonometry,* respectively, within the topic *trigonometry.* The letter D refers to the cognitive demand category *demonstrate understanding.* To support this coding I have included the assessment item in Figure 23.

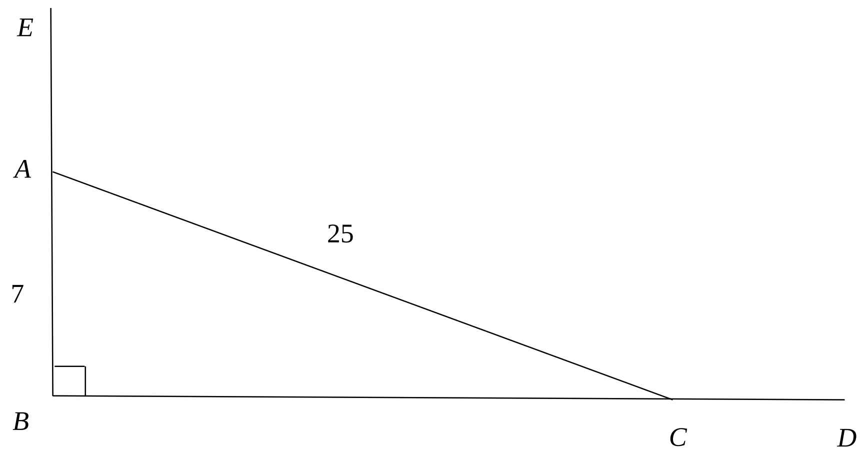
Below is a picture of $\triangle ABC$:



a. Draw a triangle *DEF* which is similar (but not congruent) to $\triangle ABC$.
b. How do $\frac{|DE|}{|DF|}$ and $\frac{|AB|}{|AC|}$ compare? Explain.
c. When $\angle B$ is a right angle, the ratio |AB|:|AC| is called the cosine of $\angle A$ while the ratio |BC|:|AC| is called the sine of $\angle A$. Why do these ratios depend only on $\angle A$?
d. The ratios in part (c) make sense whether or not $\angle B$ is a right angle but they are only given names (sine and cosine) in this special case. What is special about the case where *B* is a right angle?

Figure 23: "Defining Trigonometric Ratios" Assessment Item from Illustrative Mathematics [12]

The SMCF standard 2.4, "use of trigonometric ratios (sine, cosine, and tangent) of acute angles to calculate unknown sides and angles in right-angled triangles" [23] is similar in topic to the CCSSM standard HSG.SRT.C.6, but as the assessment item in Figure 24 shows, differs in cognitive demand. The assessment item in Figure 24 does not require that students first develop the relationship between similar triangles and the trigonometric ratios of sine, cosine, and tangent, but rather requires that students recall such ratios to complete the exercise in a procedural manner.



In the diagram, $A\hat{B}C = 90°, AB = 7\ cm, AC = 25\ cm$ and BAE and BCD are straight lines.

(a) Showing your working clearly, explain why $BC = 24\ cm$.

(b) Express as a fraction

(i) $\sin B\hat{C}A$,

(ii) $\tan A\hat{C}D$,

(iii) $\cos E\hat{A}C$.

Figure 24: Assessment Item 9 from *New Elementary Mathematics Syllabus D, Book 4B* (p. 85) [14]

CHAPTER 5: CONCLUSIONS

First, it is important to mention that a thorough analysis and comparison of two nation's educational standard documents alone will not ensure their success if one document is transferred to and implemented in another country. Additional factors such as teacher education and recruitment, social, cultural, economic, and geographic influences can have significant impacts on the educational achievement of a nation's youth.

In addition, for both standards documents, there is currently a limited number of aligned assessment items and curriculum materials on which to base an analysis. With the ongoing revision of the Singapore syllabi and the continual development of assessment items that align with the CCSSM, the results of the SEC content analysis may prove to be different as additional assessment data becomes available.

Despite these factors, based on the available assessment items and the results of SEC content analysis, it is clear that in terms of mathematics topics the CCSSM and SMCF are similar. Nearly all the mathematics topics that are present in the SMCF are present in the CCSSM. In the absence of a standard in a particular grade level, an analogous standard can usually be found within one surrounding grade level.

In terms of cognitive demand, SEC content analysis results show that the CCSSM exhibit higher percentages of standards that require more advanced levels of cognitive demand (i.e., *demonstrate understanding, conjecture/analyze, solve non-routine problems*) when compared with the standards of the SMCF. Conversely, the standards of the SMCF syllabi documents exhibit higher percentages of standards that require lower levels of cognitive demand (i.e., *memorize* and *perform procedures*). For example, the results of the SEC analysis show that for the number and algebra conceptual category/content strand, the combined percentage of standards that require students to either *memorize* or *perform procedures* is 83.9% and 68.6% for the SMCF and the CCSSM, respectively. In the geometry conceptual category/content strand, an even larger difference can be seen in comparing the percentages of standards requiring either of the aforementioned levels of cognitive demand with percentages being 94.7% and 31.3% for the SMCF and CCSSM, respectively.

In examining assessment items that align with specific standards, we are able to reinforce our conclusions regarding the levels of cognitive demand required by the standards documents. In the assessment items from *New Elementary Mathematics Syllabus D*, we see that these assessment items are most often procedural in nature and do not often require students to expound on the underlying mathematical concepts. In contrast to this, Illustrative Mathematics assessment items are often multi-step tasks (as seen in Figures 11, 15, 19, 21) that incorporate various level of cognitive demand and often guide students from basic to advanced levels of understanding.

The standards of the CCSSM are often stated with a clearer expectation of what students should know and be able to do; standards often begin with words such as "understand", "represent", "develop", "apply", and "interpret" whereas the standards in the SMCF occur as bulleted topic lists most often without clear indication of the expected student outcomes of the standards. Further research and classroom observation would be necessary to determine the meaning of standards in the SMCF in practice.

In addition, as we previously mentioned, the Singapore GCE Mathematics O-Level Syllabus is not the minimum requirement for students in terms of secondary mathematics content coverage, but rather the minimum set of mathematics standards for which the majority of Singaporean students complete courses. So the Singapore GCE Mathematics O-Level Syllabus is a standards document that is above the minimum level of expectation for all Singaporean students. In comparing the CCSSM and SMCF, we have determined that, in terms of mathematics topics and cognitive demand, the CCSSM either aligns with, or in some cases exceeds, the expectations of the SMCF O-Level Syllabus.

Although the CCSSM are still being implemented in many states, and curriculum materials and teacher resources are still being developed, we can be assured that the CCSSM present a framework that aligns, in terms of mathematics topics and cognitive demand, with the mathematics standards of the SMCF.

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APPENDIX A: AUTHORS OF THE STRANDS OF MATHEMATICAL PROFICIENCY

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Roger Howe	Mathematics	Yale University
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APPENDIX B: SEC K-12 MATHEMATICS TAXONOMY

SEC K-12 Mathematics Taxonomy

100	Nbr. sense /Properties/ Relationships	900	Data Displays
200	Operations	1000	Statistics
300	Measurement	1100	Probability
400	Consumer Applications	1200	Analysis
500	Basic Algebra	1300	Trigonometry
600	Advanced Algebra	1400	Special Topics
700	Geometric Concepts	1500	Functions
800	Advanced Geometry	1600	Instructional Technology

Other Coding Conventions

Topics:

0	All
999	Out of Subject Area

Cognitive Demands:

B	Memorize
C	Perform Procedures
D	Demonstrate Understanding
E	Conjecture/Analyze
F	Solve Non-Routine Problems
Z	Non-Specific Cognitive Demand

APPENDIX C: SEC K-12 MATHEMATICS TOPIC AND SUBTOPIC TAXONOMY

K-12 Mathematics Taxonomy

100	**Nbr. sense /Properties/ Relationships**
101	Place value
102	Whole numbers and Integers
103	Operations
104	Fractions
105	Decimals
106	Percents
107	Ratio and proportion
108	Patterns
109	Real and/or Rational numbers
110	Exponents and scientific notation
111	Factors, multiples, and divisibility
112	Odd/even/prime/composite/square numbers
113	Estimation
114	Number Comparisons (order, magnitude, relative size, inverse, opposites, equivalent forms, scale or number line)
115	Order of operations
116	Computational Algorithms
117	Relationships between operations
118	Number Theory (e.g. base-ten and non-base-ten systems)
119	Mathematical properties (e.g., distributive property)
190	Other
200	**Operations**
201	Add/subtract whole numbers and integers
202	Multiply whole numbers and integers
203	Divide whole numbers and integers
204	Combinations of operations on whole numbers or integers
205	Equivalent and non-equivalent fractions
206	Add/subtract fractions
207	Multiply fractions
208	Divide fractions
209	Combinations of operations on fractions
210	Ratio and proportion
211	Representations of fractions
212	Equivalence of decimals, fractions, and percents
213	Add/ subtract decimals
214	Multiply decimals
215	Divide decimals
216	Combinations of operations on decimals
217	Computing with percents
218	Computing with exponents and radicals
290	Other

300	**Measurement**
301	Use of measuring instruments
302	Theory (arbitrary, standard units and unit size)
303	Conversions
304	Metric (SI) system
305	Length and perimeter
306	Area and volume
307	Surface Area
308	Direction, Location, Navigation
309	Angles
310	Circles (e.g., pi, radius, area)
311	Mass (weight)
312	Time and temperature
313	Money
314	Derived measures (e.g., rate and speed)
315	Calendar
316	Accuracy and Precision
390	Other
400	**Consumer Applications**
401	Simple interest
402	Compound interest
403	Rates (e.g., discount and commission)
404	Spreadsheets
490	Other
500	**Basic Algebra**
501	Absolute value
502	Use of variables
503	Evaluation of formulas, expressions, and equations
504	One-step equations
505	Coordinate Planes
506	Patterns
507	Multi-step equations
508	Inequalities
509	Linear and non-linear relations
510	Rate of change/slope/line
511	Operations on polynomials
512	Factoring
513	Square roots and radicals
514	Operations on radicals
515	Rational expressions
516	Multiple representations
590	Other

K-12 Mathematics Taxonomy

600	**Advanced Algebra**
601	Quadratic equations
602	Systems of equations
603	Systems of inequalities
604	Compound Inequalities
605	Matrices and determinants
606	Conic sections
607	Rational, negative exponents/radicals
608	Rules for exponents
609	Complex numbers
610	Binomial theorem
611	Factor/remainder theorem
612	Field properties of real number system
613	Multiple representations
690	Other
700	**Geometric Concepts**
701	Basic terminology
702	Points, lines, rays, segments, and vectors
703	Patterns
704	Congruence
705	Similarity
706	Parallels
707	Triangles
708	Quadrilaterals
709	Circles
710	Angles
711	Polygons
712	Polyhedra
713	Models
714	3-D relationships
715	Symmetry
716	Transformations (e.g., flips or turns)
717	Pythagorean Theorem
790	Other
800	**Advanced Geometry**
801	Logic, reasoning, and proofs
802	Loci
803	Spheres, cones, and cylinders
804	Coordinate Geometry
805	Vectors
806	Analytic Geometry
807	Non-Euclidean Geometry
808	Topology
890	Other

900	**Data Displays**
901	Summarize data in a table or graph
902	Bar graph and histograms
903	Pie charts and circle graphs
904	Pictographs
905	Line graphs
906	Stem and Leaf plots
907	Scatter plots
908	Box plots
909	Line plots
910	Classification and Venn diagrams
911	Tree diagrams
990	Other
1000	**Statistics**
1001	Mean, median, and mode
1002	Variability, standard deviation, and range
1003	Line of best fit
1004	Quartiles and percentiles
1005	Bivariate distribution
1006	Confidence intervals
1007	Correlation
1008	Hypothesis testing
1009	Chi Square
1010	Data Transformation
1011	Central Limit Theorem
1090	Other
1100	**Probability**
1101	Simple probability
1102	Compound probability
1103	Conditional probability
1104	Empirical probability
1105	Sampling and Sample spaces
1106	Independent vs. dependent events
1107	Expected value
1108	Binomial distribution
1109	Normal curve
1190	Other
1200	**Analysis**
1201	Sequences and series
1202	Limits
1203	Continuity
1204	Rates of change
1205	Maxima, Minima, and Range
1206	Differentiation
1207	Integration
1290	Other

K-12 Mathematics Taxonomy

1300	**Trigonometry**
1301	Basic ratios
1302	Radian measure
1303	Right triangle trigonometry
1304	Law of Sines and Cosines
1305	Identities
1306	Trigonometric equations
1307	Polar coordinates
1308	Periodicity
1309	Amplitude
1390	Other
1400	**Special Topics**
1401	Sets
1402	Logic
1403	Mathematical induction
1404	Linear programming
1405	Networks
1406	Iteration and recursion
1407	Permutation combinations
1408	Simulations
1409	Fractals
1490	Other
1500	**Functions**
1501	Notation
1502	Relations
1503	Linear
1504	Quadratic
1505	Polynomial
1506	Rational
1507	Logarithmic
1508	Exponential
1509	Trigonometric and circular
1510	Inverse
1511	Composition
1590	Other
1600	**Instructional Technology**
1601	Use of calculators
1602	Use of graphing calculators
1603	Use of computers and internet
1604	Computer programming
1605	Use of Spreadsheets
1690	Other

APPENDIX D: Cognitive Demand Categories for Mathematics

Cognitive Demand Categories for Mathematics

B	C	D	E	F
Memorize Facts, Definitions, Formulas	**Perform Procedures**	**Demonstrate Understanding of Mathematical Ideas**	**Conjecture, Analyze, Generalize, Prove**	**Solve Non-Routine Problems / Make Connections**
Recite basic mathematical facts	Use numbers to count, order, denote	Communicate mathematical ideas	Determine the truth of a mathematical pattern or proposition	Apply and adapt a variety of appropriate strategies to solve non-routine problems
Recall mathematics terms and definitions	Do computational procedures or algorithms	Use representations to model mathematical ideas	Write formal or informal proofs	Apply mathematics in contexts outside of mathematics
Recall formulas and computational procedures	Follow procedures / instructions	Explain findings and results from data analysis strategies	Recognize, generate or create patterns	Apply to real world situations
	Solve equations/formulas/ routine word problems	Develop/explain relationships between concepts	Find a mathematical rule to generate a pattern or number sequence	Synthesize content and ideas from several sources
	Organize or display data	Show or explain relationships between models, diagrams, and/or other representations	Make and investigate mathematical conjectures	
	Read or produce graphs and tables		Identify faulty arguments or misrepresentations of data	
	Execute geometric constructions		Reason inductively or deductively	

GCE Mathematics Ordinary Level Syllabus 4016	Common Core State Standards for Mathematics
Numbers and Algebra	
1.1 Numbers and the four operations Include:	
• Primes and prime factorization	• **CCSSM.4.OA.B.4**
• Finding HCF and LCM, squares, cubes, square roots and cube roots by prime factorization	• **CCSSM.6.NS.B.4, CCSSM.8.EE.A.2**
• Negative numbers, integers, rational numbers, real numbers and their four operations	• **CCSSM.7.NS.A.3**
• Calculations with the use of a calculator	• **(CCSSM High School Number and Quantity)**
• Representation and ordering of numbers on the number line	• **CCSSM.6.NS.C.6.B**
• Use of the symbols $<, >, \leq, \geq$	• **CCSSM.6.EE.B.8, CCSSM.7.EE.B.4.B**
• Approximation and estimation (including rounding off numbers to a required number of decimal places or significant figures, estimating the results of computation, and concepts of rounding and truncation errors)	• **CCSSM.3.OA.D.8, CCSSM.4.NBT.A.3**
• Examples of very large and very small numbers such as mega/million (10^6), giga/billion (10^9), terra/trillion (10^{12}), micro (10^{-6}), nano (10^{-9}), and pico (10^{-12})	• **CCSSM.8.EE.A.3**
• Use of standard form $A \times 10^n$, where n is an integer, and $1 \leq A \leq 10$	• **CCSSM.8.EE.A.4**
• Positive, negative, zero and fraction indices	• **CCSSM.8.EE.A.1, CCSSM.HSN.RN.A.1**
• Laws of indices	• **CCSSM.8.EE.A.1**

1.2 Ratio, rate and proportion Include:	
• Ratios involving rational numbers	• **CCSSM.6.RP.A.2**
• Writing a ratio in its simplest form	
• Average rate	
• Map scales (distance and area)	• **CCSSM.7.G.A.1**
• Direct and inverse proportion	
• Problems involving ratio, rate and proportion	• **CCSSM.6.RR.A.3**
1.3 Percentage Include:	
• Expressing one quantity as a percentage of another	• **CCSSM.6.RP.A.3.C**
• Comparing two quantities by percentage	
• Percentages greater than 100%	
• Increasing/decreasing a quantity by a given percentage	• **CCSSM.7.RP.A.3**
• Reverse percentages	
• Problems involving percentages	• **CCSSM.7.RP.A.3**
1.4 Speed Include:	
• Concepts of speed, uniform speed and average speed	• **CCSSM.RP.A.3.B**
• Conversion of units (e.g. km/h to m/s)	• **CCSSM.5.MD.A.1, CCSSM.6.RP.A.3.D**
• Problems involving speed, uniform speed and average speed	• **CCSSM.RP.A.3.B**

1.5 Algebraic representation and formulae
Include:
• Using letters to represent numbers
• Interpreting notations:
  • ab as $a \times b$
  • $\frac{a}{b}$ as $a \div b$
  • a^2 as $a \times a, a^3$ as $a \times a \times a, \ldots$
  • $3y$ as $y + y + y$ or $3 \times y$
  • $3(x + y)$ as $3 \times (x + y)$
  • $\frac{3+y}{5}$ as $(3 + y) \div 5$ or $\frac{1}{5} \times (3 + y)$
• Evaluation of algebraic expressions and formulae
• Translation of simple real-world situations into algebraic expressions
• Recognizing and representing number patterns (including finding an algebraic expressions for the *n*th terms)
	• **CCSSM.6.EE.B.6**
	
	• **CCSSM.7.EE.A.1**
	• **CCSSM.7.EE.B.4**

1.6 Algebraic manipulation Include: • Addition and subtraction of linear algebraic expressions • Simplification of linear algebraic expressions, e.g. $-2(3x - 5) + 4x$ $\frac{2x}{3} - \frac{3(x-5)}{2}$ • Factorization of linear algebraic expressions of the form	• **CCSSM.7.EE.A.1** • **CCSSM.7.EE.A.1**

• $ax + by$ (where a is a constant) • $ax + bx + kay + kby$ (where a, b, and k are constants)	
• Expansion of the product of algebraic expressions • Changing the subject of a formula • Finding the value of an unknown quantity in a given formula	• **CCSSM.HSA.APR.A.1** • **CCSSM HSA.CED.A.4**

• Recognizing and applying the special products
  • $(a \pm b)^2 = a^2 \pm 2ab + b^2$
  • $a^2 - b^2 = (a+b)(a-b)$
	• **CCSSM.HSA.APR.C.4**

• Factorization of algebraic expressions of the form
  • $a^2x^2 - b^2y^2$
  • $a^2 \pm 2ab + b^2$
  • $ax^2 + bx + c$
	• **CCSSM.HSA.SSE.B.3.A**

• Multiplication and division of simple algebraic functions, e.g. $\left(\frac{3a}{4b^2}\right)\left(\frac{5ab}{3}\right)$ $\frac{3a}{4} \div \frac{9a^2}{10}$	• **(+) CCSSM.HSA.APR.D.7**
• Addition and subtraction of algebraic fractions with linear or quadratic denominator, e.g. $\frac{1}{x-2} + \frac{2}{x-3}$	• **(+) CCSSM.HSA.APR.D.7**

$$\frac{1}{x^2-9}+\frac{2}{x-3}$$ $$\frac{1}{x-3}+\frac{2}{(x-3)^2}$$	
1.7 Functions and graphs Include:	
• Cartesian coordinates in two dimensions	• **CCSSM.5.G.A.1**
• Graph of a set of ordered pairs	• **CCSSM.6.NS.C.8, CCSSM.8.F.A.1**
• Linear relationships between two variables (linear functions)	• **CCSSM.HSF.BF.A.1**
• The gradient of a linear graph as the ratio of the of the vertical change to the horizontal change (positive and negative gradients)	• **CCSSM.8.EE.C.8**
• Graphs of linear equations in two unknowns	• **CCSSM.8.EE.C.8**

• Graphs of quadratic functions and their properties
  ◦ Positive or negative coefficent of x^2
  ◦ Maximum and minimum points
  ◦ Symmetry
	• **CCSSM.HSF.IF.C.7.A., CCSSM.HSF.IF.C.8.A**

• Sketching the graphs of quadratic functions given in the form
  ◦ $y=\pm(x-p)^2+q$
  ◦ $y=\pm(x-a)(x-b)$
	• **CCSSM.HSF.IF.C.7**

• Graphs of function of the form $y=ax^n$ where $n=-2,-1,0,1,2,3$, and simple sums of not more than three of these	• **CCSSM.HSF.IF.C.7.E**
• Graphs of exponential functions $y=ka^x$where a is a positive integer	• **CCSSM.HSF.IF.C.7.E**

• Estimation of gradients of curves by drawing tangents	
1.8 Solutions of equations and inequalities Include:	
• Solving linear equations in one unknown (including fractional coefficients)	• **CCSSM.8.EE.C7.B**
• Solving simple fractional equations that can be reduced to linear equations, e.g. $\frac{x}{3}+\frac{x-2}{4}=3$ $\frac{3}{x-2}=6$	• **CCSSM.HSA.REI.A.2**
• Solving simultaneous linear equations in two unknowns by ◦ Subsitution and elimination methods ◦ Graphical method	• **CCSSM.HSA.REI.C.6**
• Solving quadratic equations in one unknown by ◦ Factorization ◦ Use of formula ◦ Completing the square for $y=x^2+px+q$ ◦ Graphical methods	• **CCSSM.HSA.REI.B.4, CCSSM.HSA.SSE.B.3.A**
• Solving fractional equations that can be reduced to quadratic equations, e.g. $\frac{6}{x+4}=x+3$ $\frac{1}{x-2}+\frac{2}{x-3}=5$	• **CCSSM.HSA.REI.A.2, CCSSM.HSA.REI.B.4**

• Formulating equations to solve problems	• **CCSSM.HSA.CED.A.1**
• Solving linear equations in one unknown, and representing the solution set on the number line	• **CCSSM.8.EE.B.5**
1.9 Applications of mathematics in practical solutions Include:	

• Problems derived from practical situations such as
  ◦ Utilities bills
  ◦ Hire-purchase
  ◦ Simple interest and compound interest
  ◦ Money exchange
  ◦ Profit and loss
  ◦ Taxation
	• **CCSSM.7.RR.A.3**

• Use of data from tables and charts • Interpretation and use of graphs in practical situations • Drawing graphs from given data • Distance-time and speed-time graphs	• **CCSSM.8.EE.B.5**
Exclude the use of the terms “percentage profit” and “percentage loss”.	
1.10 Set language and notation Include:	

• Use of set laguage and the following notation:
  ◦ Union of A and B — $A \cup B$
  ◦ Intersection A and B — $A \cap B$
  ◦ Number of elements in set A — $n(A)$
  ◦ “…is an element of…” — $\in$	

• "…is not an element of…" $\notin$	
• Complement of set A A'	
• The empty set $\emptyset$	
• Universal set ξ	
• A is a subset of B $A \subseteq B$	
• A is a proper subset of B $A \subset B$	
• A is not a subset of B $A \nsubseteq B$	
• A is not a proper subset of B $A \not\subset B$	
• Union and intersection of two sets	• **CCSSM.HSS.CP.A.1**
• Venn diagrams	
Exclude:	
• Use of $n(A \cup B) = n(A) + n(B) - n(A \cap B)$	
• Cases involving three or more sets	
1.11 Matrices	
Include:	
• Display of information in the form of a matrix of any order	• **(+) CCSSM.HSN.VM.C.6**
• Interpreting the data in a given matrix	• **(+) CCSSM.HSN.VM.C.6**
• Product of a scalar quantity and a matrix	• **(+) CCSSM.HSN.VM.C.7**
• Problems involing the calucation of the sum and product (where appropriate) of two matrices	• **(+) CCSSM.HSN.VM.C.7**
Exclude:	
• Matrix representation of geometrical trasnformations	
• Solving simulataneous linear equations using the inverse matrix method	

GCE Mathematics Ordinary Level Syllabus 4016	Common Core State Standards for Mathematics
Geometry and Measurement	**Geometry**
2.1 Angles, triangles, and polygons Include:	
• Right, acute, obtuse and reflex angles, complementary and supplemenatary angles, vertically opposite angles, adjacent angles in a straight line, adjacent angles at a point, interior and exterior angles	• **CCSSM.7.G.B.5, CCSSM.HSG.CO.C.9**
• angles formed by two parallel lines and a transversal: corresponding angles, alternate angles, interior angles	• **CCSSM.8.G.A.5, CCSSM.HSG.CO.C.9**
• properties of triangles and special quadrilaterals	• **CCSSM.HSG.SRT.B.4, CCSSM.HSG.SRT.B.5**
• classifying special quadrilaterals on the basis of their properties	• **CCSSM.5.G.B.4**
• angles sum of interior and exterior angles of any convex polygon	
• properties of regular pentagon, hexagon, octagon and decagon	
• properties of perpendicular bisectors of line segments and angle bisectors	• **CCSSM.HSG.SRT.B.5**
• construction of simple geometrical figures from given data (including perpendicular bisectors and angle bisectors) using compasses, ruler, set squares and protractor, where appropriate	• **CCSSM.HSG.SRT.A.2**
2.2 Congruence and similarity Include:	
• congruent figures and similar figures	• **CCSSM.HSG.SRT.B.5**
• properties of similar polygons:	• **CCSSM.HSG.SRT.A.2**

• corresponding angles are equal • corresponding sides are proportion • enlargement and reduction of a plane figure by a scale factor	• **CCSSM.HSG.SRT.A.1**
• scale drawings	• **CCSSM.7.G.A.1**
• determining whether two triangles are congruent • congruent • similar • ratio of areas of similar plane figures • ratio of volumes of similar solids	• **CCSSM.HSG.SRT.B.5**
• solving simple problems involving similarity and congruence	• **CCSSM.HSG.SRT.B.5**
2.3 Properties of circles Include:	
• symmetry properties of circles: • equal chords are equidistant from the centre • the perpendicular bisector of a chord passes through the centre • tangents from an external point are equal in length • the line joining an external point to the centre of the circle bisects the angles between the tangents	• **CCSSM.HSG.C.A.2**
• angle properties of circles: • angle in a semicircle is a right angle • angle between tangent and radius of a circle is a right angle	• **CCSSM.HSG.C.A.2**

• angle at the centre is twice the angle at the circumference • angles in the same segment are equal • angles in opposite segments are supplementary	
2.4 Pythagoras' theorem and trigonometry Include: • use of Pythagoras' theorem • determining whether a triangle is right-angled given the lengths of three sides • use of trigonometric ratios (sine, cosine, and tangent) of acute angles to calculate unknown sides and angles in right-angled triangles • extending sine and cosine to obtuse angles • use of the formula $\frac{1}{2}ab\sin C$ for the area of a triangle • use of sine rule and cosine rule for any triangle • problems in 2 and 3 dimensions including those involving angles of elevation and depression and bearings Exclude calculation of the angle between two planes or of the angle between a straight line and a plane.	• **CCSSM.8.G.B.7, CCSSM.8.G.B.8** • **CCSSM.HSG.SRT.C.6** • **(+) CCSSM.HSG.SRT.D.9** • **(+) CCSSM.HSG.SRT.D.10, (+) CCSSM.SRT.D.11**
2.5 Mensuration Include: • area of parallelogram and trapezium • problems involving perimeter and area of composite plane figures (including triangle and circle)	• **CCSSM.7.G.B.6** • **CCSSM.7.G.B.6** • **CCSSM.7.G.B.6, CCSSM.HSG.GMD.A.3**

• volume and surface area of cube, cuboid, prism, cylinder, pyramid, cone and sphere • conversion between cm^2 and m^2, and between cm^3 and m^3 • problems involving volume and surface area of composite solids • arc length and sector area as fractions of the circumference and area of a circle • area of a segment • use of radian measure of angle (including conversion between radians and degrees) • problems involving the arc length, sector area of a circle and area of a segment **2.6 Coordinate geometry** Include: • finding the gradient of a straight line given the coordinates of two points on it • finding the length of a line segment given the coordinate of its end points • interpreting and finding the equation of a straight line graph in the form $y = mx + c$ • geometric problems involving the use of coordinates Exclude: • condition for two lines to be parallel or perpendicular • mid-point of line segment • finding the area of quadrilateral given its vertices	• **CCSSM.7.G.B.6** • **CCSSM.HSG.C.B.5** • **CCSSM.HSG.C.B.5** • **CCSSM.HSG.C.B.5** • **CCSSM.HSG.GPE.B.7** • **CCSSM.8.F.A.3** • **CCSSM.HSG.GPE.B.4**

2.7 Vectors in two dimensions Include: • use of notations: $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$, $\boldsymbol{a}$, $\lvert\overrightarrow{AB}\rvert$, and $\lvert\boldsymbol{a}\rvert$ • directed line segments • translation by a vector • position vectors • magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$ • use of sum and difference of two vectors to express given vectors in terms of two coplanar vectors • multiplication of a vector by a scalar • geometric problems involving the use of vectors Exclude: • expressing a vector in terms of a unit vector • mid-point of line segment • solving vector equations with two unknown parameters	• **(+) CCSSM.HSN.VM.A.1** • **(+) CCSSM.HSN.VM.A.1** • **(+) CCSSM.HSN.VM.A.1** • **(+) CCSSM.HSN.VM.B.4** • **(+) CCSSM.HSN.VM.B.5**

APPENDIX G: SEC CODING OF STANDARDS: GRADE 7/SECONDARY 1

Common Core State Standards for Mathematics		**Singapore O-Level Mathematics Syllabus**	
The Number System	**SEC Code(s)**	**Number and Algebra**	**SEC Code(s)**
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. **NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have the sum of 0. Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers	114D, 209D, 207D, 208D, 209D, 501D	**N1 Numbers and their operations** **1.1** primes and prime factorization **1.2** finding highest common factor (HCF), lowest common multiple (LCM), squares, cubes, square roots, and cube roots by prime factorization **1.3** negative numbers, integers, rational numbers, real numbers, and their four operations **1.4** calculations with calculator **1.5** representation and ordering of numbers on the number line **1.6** use of $<, >, \leq, \geq$ **1.7** approximation and estimation (including rounding off numbers to a required number of decimal places or significant figures and estimating the results of computation) **N.2 Ratio and Proportion** **2.1** ratios involving rational numbers **2.2** writing a ratio in its simplest form **2.3** problems involving ratio	 111B, 112B 110C, 111C, 112C, 513C 109C, 204C, 204D 1601C 114D 508D 105C, 113C 107C, 109C 107C, 205C 210C

on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.		**N.3 Percentage** **1.1** expressing one quantity as a percentage of another **1.2** comparing two quantities by percentage **1.3** percentages great than 100%	 106C, 107C, 217C 106C, 212
NS.A.2 Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers.	119D, 202D, 207D, 208D, 212D	**1.4** increasing/decreasing a quantity by a given percentage (including concept of percentage point) **3.5** reverse percentages **3.6** problems involving percentages	106C 106C 106C, 217C 106C, 217C
a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1)=1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.		**N.4 Rate and speed** **4.1** concepts of average rate, speed, constant speed and average speed **4.2** conversion of units (e.g. km/h to m/s) **4.3** problems involving rate and speed	 314B, 1001B 303C, 304C 314C
b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisors) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right)=\frac{(-p)}{q}=\frac{p}{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts.		**N.5 Algebraic expressions and formulae** **5.1** using letters to represent numbers **5.2** interpreting notations: • ab as $a\times b$ • $\frac{a}{b}$ as $a\div b$ • a^2 as $a\times a, a^3$ as $a\times a\times a, \ldots$ • $3y$ as $y+y+y$ or $3\times y$ • $3(x+y)$ as $3\times(x+y)$ • $\frac{3+y}{5}$ as $(3+y)\div 5$ or $\frac{1}{5}\times(3+y)$ **5.3** evaluation of algebraic expressions and formulae	 502D 117D, 516D 503C

c. Apply properties of operations as strategies to multiply and divide rational numbers.		**5.4** translation of simple real-world situations into algebraic expressions	502F, 503F, 507F
d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.		**5.5** recognizing and representing patterns/relationships by finding an algebraic expression for the *n*th term	506D
		5.6 addition and subtraction of linear expressions	509C
NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.	109C, 109F	**5.7** simplification of linear expressions such as $-2(3x-5)+4x$ $\frac{2x}{3}-\frac{3(x-5)}{2}$	503C, 509C, 511C, 515C
Ratios and Proportional Relationships			
Analyze proportional relationships and use them to solve real-world and mathematical problems.		**5.8** use brackets and extract common factors	512C
		N.6 Functions and graphs	
RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.	210C, 302C, 305C, 306C	**6.1** Cartesian coordinates in two dimensions	505D
		6.2 graph of a set of ordered pairs as a representation of a relationship between two variables	505D
RP.A.2 Recognize and represent proportional relationships between quantities.	210E, 212D, 505D	**6.3** linear functions	1503C
		6.4 graphs of linear functions	1503C
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.		**6.5** the gradient of a linear graph as the ratio of the vertical change to the horizontal change (positive and negative gradients)	510D
		N.7 Equations and inequalities	
		7.1 concepts of equation and inequality	503D, 508D
b. Identify the contrast of proportionality (unit rate) in tables, graphs, equations,		**7.2** solving linear equations in one variable	509C

diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. d. Explain what a point (x, y) on a graph of a proportional relationship means in terms of the situation, with special attention to the points (0,0) and $(1, r)$ where r is a unit rate.		**7.3** solving inequalities in the form $ax \leq b, ax \geq b, ax < b$ and $ax > b$, where a and b are integers	508C
		7.4 solve simple fractional equations that can be reduced to linear equations such as $$\frac{x}{3} + \frac{x-2}{4} = 3$$ $$\frac{3}{x-2} = 6$$	503C, 507C, 511C, 515C
RP.A.3 Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	210C, 217C, 401C, 403C, 490C, 507C	**7.5** formulating a linear equation in one variable to solve problems	502C, 509C
		N.10 Problems in real-world contexts	
		10.1 solving problems based on real-world contexts: • in everyday life (including travel plans, transport schedules, sports and games, recipes, etc.) • involving personal and household finance (including simple interest, taxation, instalments, utilities bills, money exchange, etc.)	313F, 315F, 401F, 502F, 507F, 590F
Expressions and Equations			
Use properties of operations to generate equivalent expressions.			
EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	503C, 509C, 512C		
EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.	516D	**10.2** interpreting and analyzing data from tables and graphs including distance-time and time-speed graphs	314E, 901E
		10.3 interpreting the solution in the context of the problem	590E
		10.4 identifying assumptions made and the limitations of the solution	590E

Solve real-life and mathematical problems using numerical and algebraic expressions and equations. **EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess computation and estimation strategies.	113C, 204C, 209C, 212C, 216C, 507F		
EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q and r	502F, 505F, 508F		

are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.			
Geometry		**Geometry and Measurement**	
Draw, construct, and describe geometrical figures and describe the relationships between them.		**G.1 Angles, triangles, and polygons**	
		1.1 right, acute, obtuse, and reflex angles	701B, 710B
		1.2 vertically opposite angles, angles on a straight line, angles at a point	701B, 710B
G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	305C, 306C, 790C	**1.3** angles formed by two parallel lines and a transversal: corresponding angles, alternate angles, interior angles	701B, 706B, 710B
G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	301C, 707D, 1603C	**1.4** properties of triangles, special quadrilateral and regular polygons (pentagon, hexagon, octagon, and decagon), including symmetry properties	707B, 708B, 711B, 715B
		1.5 classifying special quadrilateral on the basis of their properties	708D
		1.6 angle sum of interior and exterior angles of any convex polygon	710B, 711B, 710C, 711C
G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	714D, 790C	**1.7** properties of perpendicular bisectors of line segments and angle bisectors	701B, 702B
		1.8 construction of simple geometrical figures from given data (including perpendicular bisectors) using compasses, ruler, set squares and protractors, where appropriate	301C, 790C

Solve real-life and mathematical problems involving angle measures, area, surface area, and volume.		**G.5 Mensuration**	
		5.1 area of parallelogram and trapezium	306C, 708C, 711C
G.B.4 Know the formulas for area and circumference of a circle and use them to solve problems; give an informal derivation of the relationships between the circumference and area of a circle.	310B, 801E	**5.2** problems involving perimeter and area of plane figures **5.3** volume and surface area of prism and cylinder	305C, 306C, 790C 306C, 307C, 712C, 803C
G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	710B, 710C	**5.4** conversion between cm^2 and m^2 and between cm^3 and m^3 **5.5** problems involving volume and surface area of composite solids	303C, 304C 306C, 307C, 712C
G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	306F, 307F, 707F, 708F, 711F, 712F	**G.8 Problems in real-world contexts** **8.1** solving problems in real-world contexts (including floor plans, surveying, navigation, etc.) using geometry **8.2** interpreting the solutions in the context of the problem **8.3** identifying the assumptions made and the limitations of the solution	 790F 790F 790F
Statistics and Probability		**Statistics and Probability**	
Use random sampling to draw inferences about a population.		**S.1 Data analysis**	
SP.A.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample size is	1105E	**1.1** analysis and interpretation of: • tables • bar graphs • pictograms • line graphs • pie charts	901E, 902E, 903E, 904E, 905E

representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.		**1.2** purpose and uses, advantages and disadvantages of the different forms of statistical representations	990E, 1090E
SP.A.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.	1105E	**1.3** explaining why a given statistical diagram leads to misinterpretation of data	990E, 1090E
Draw informal comparative inferences about two populations.			
SP.B.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.	1002E		
SP.B.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.	1001E, 1002E		
Investigate chance processes and develop, use, and evaluate probability models.			
SP.C.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event	1101C		

occurring. Large numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.			
SP.C.6 Approximate the probability of a chance event by collecting date on the chance process that produces it and observing its long-run relative frequency, and predicates the approximate relative frequency given the probability.	1101D		
SP.C.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model assigning equal probability to all outcomes, and use the model to determine probabilities of events. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.	1190E		
SP.C.8 Find probabilities of compound events using lists, tables, tree diagrams, and simulation.	911C, 990C, 1101D,		

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language, identify the outcomes in the sample spaces which compose the event. c. Design and use a simulation to generate frequencies for compound events.	1102D, 1105D		

APPENDIX H: SEC CODING OF STANDARDS: GRADE 8/SECONDARY 2

Common Core State Standards for Mathematics		Singapore O-Level Mathematics Syllabus	
The Number System	**SEC Code(s)**	**Number and Algebra**	**SEC Code(s)**
Know that there are numbers that are not rational, and approximate them by rational numbers.		**N.2 Ratio and Proportion**	
		2.4 map scales (distance and area)	107C, 306C, 308C
NS.A.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	109C, 113C, 190B	**2.5** direct and inverse proportion	210C
		N.5 Algebraic expressions and formulae	
		5.9 expansion of the product of algebraic expressions	503C, 511C
		5.10 changing the subject of a formula	209C, 502C, 515C
NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g. π^2).	113C, 114C, 109C	**5.11** find the value of an unknown quantity in a given formula	502C, 507C
		5.12 use of: • $(a+b)^2 = a^2 + 2ab + b^2$ • $(a-b)^2 = a^2 - 2ab + b^2$ • $a^2 - b^2 = (a+b)(a-b)$	511C, 512C
Expressions and Equations		**5.13** factorization of linear expressions of the form $ax + bx + kay + by$	512C
Work with radicals and integer exponents.			
EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.	110B, 218C	**5.14** factorization of quadratic expressions $ax^2 + bx + c$	512C, 601C
		5.15 multiplication and division of simple algebraic fractions such as	207C, 208C, 511C, 515C

EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	190B, 503C 513C	$\left(\frac{3a}{4b^2}\right)\left(\frac{5ab}{3}\right)$ $\frac{3a}{4} \div \frac{9a^2}{10}$	
EE.A.3 Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.	110C, 113C	**5.16** addition and subtraction of algebraic fractions with linear or quadratic denominator such as $\frac{1}{x-2} + \frac{2}{x-3}$	206C, 515C
EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.	105C, 110C, 1603D	$\frac{1}{x^2-9} + \frac{2}{x-3}$ $\frac{1}{x-3} + \frac{2}{(x-3)^2}$	
Understand the connections between proportional relationships, lines, and linear equations.		**N.6 Functions and graphs** **6.6** quadratic functions $y = ax^2 + bx + c$	1504C
		6.7 graphs of quadratic functions and their properties: • positive or negative coefficient of x^2 • maximum and minimum points • symmetry	505C, 1504C
EE.B.5 Graph proportional relationships, interpreting the unit rate as the slopes of the graph. Compare two different proportional relationships represented in different ways.	505C, 510D, 516D		
EE.B.6 Use similar triangles to explain why the slope m is the same between any two	505D, 510D,	**N.7 Equations and inequalities** **7.6** graphs of linear equations in two variables ($ax + by = c$)	505C

distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	705D, 1503E	**7.7** solving simultaneous linear equations in two variable by: • substitution and elimination methods • graphical method	602C
		7.8 solving quadratic equations in one variable by factorization	512C, 601C
Analyze and solve linear equations and pairs of simultaneous linear equations.		**7.9** formulating a pair of linear equations in two variables to solve problems	502C, 602C
EE.C.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	119C, 503C, 507C, 509C	**N.10 Problems in real-world contexts** **10.1** solving problems based on real-world contexts: • in everyday life (including travel plans, transport schedules, sports and games, recipes, etc.) • involving personal and household finance (including simple interest, taxation, instalments, utilities bills, money exchange, etc.)	313F, 315F, 401F, 502F, 507F, 590F
		10.2 interpreting and analyzing data from tables and graphs including distance-time and time-speed graphs	314E, 901E
EE.C.8 Analyze and solve pairs of simultaneous linear equations. d. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs,	505C, 507C, 602E, 690F	**10.3** interpreting the solution in the context of the problem	590E
		10.4 identifying assumptions made and the limitations of the solution	590E

because points of intersection satisfy both equations simultaneously. e. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. f. Solve real-world mathematical problems leading to two linear equations in two variables.			
Functions			
Define, evaluate, and compare functions. **F.A.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set ordered pairs consisting of an input and the corresponding output.	1501C		
F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	1590D		
F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.	1503D, 1590B		

Use functions to model relationships between quantities. **F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a tables of values. **F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	510C, 510D, 1503D 1502E, 1503E, 1590E		
Geometry		**Geometry and Measurement**	
Understand congruence and similarity using physical models, transparencies, or geometry software. **G.A.1** Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length.	 716E	**G.2 Congruence and similarity** **2.1** congruent figures **2.2** similar figures **2.3** properties of similar triangles and polygons: • corresponding angles are equal • corresponding sides are proportional	 704B 705B 705B, 707B, 711B

b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.		**2.4** enlargement and reduction of a plane figure **2.5** scale drawings	790C 790C
G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	704D, 716D	**2.6** solving simple problems involving congruence and similarity **G.4 Pythagoras' theorem and trigonometry** **4.1** use of Pythagoras' theorem **4.2** determining whether a triangle is right-angled given the length of the three sides	704C, 705C 717D 707D, 717D
G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	716D	**4.3** use of trigonometric ratios (sine, cosine, and tangent) of acute angles to calculate unknown sides and angles in right-angles triangles	707C, 1301C
G.A.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	705D, 716D	**G.5 Mensuration** **5.6** volume and surface area of pyramid, cone, and sphere	 306C, 307C, 712C, 803C
G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion from similarity of triangles.	707E, 710E	**G.8 Problems in real-world contexts** **8.1** solving problems in real-world contexts (including floor plans, surveying, navigation, etc.) using geometry **8.2** interpreting the solutions in the context of the problem **8.3** identifying the assumptions made and the limitations of the solution	 790F 790F 790F

Understand and apply the Pythagorean Theorem. **G.B.6** Explain a proof of the Pythagorean Theorem and its converse. **G.B.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **G.B.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** **G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	 717E 717C, 717F 717C 306B, 803B, 803F		
Statistics and Probability		**Statistics and Probability**	
Investigate patterns of association in bivariate data. **SP.A.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	 907E, 1005E, 1090E	**S.1 Data analysis** **1.4** analysis and interpretation of: • dot diagrams • histograms • stem-and-leaf diagrams **1.5** purpose and uses, advantages and disadvantages of the different forms of statistical representations	 901E, 902E, 906E, 990E 990E, 1090E

SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **SP.A.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. **SP.A.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variable collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.	905B, 907C, 1003C 1005C 1005D	**1.6** explaining why a given statistical diagram leads to misinterpretation of data **1.7** mean, mode, and median as measures of central tendency for a set of data **1.8** purpose and use of mean, mode, and median **1.9** calculation of the mean for grouped data **S.2 Probability** **2.1** probability as a measure of chance **2.2** probability of single events (including listing all the possible outcomes in a simple chance situation to calculate the probability)	990E, 1090E 1001C 1001B 1001C 1101C 1101C

APPENDIX I: SEC CODING GEOMETRY: SINGAPORE GCE O-LEVEL SECONDARY SYLLABUS AND CCSSM

GCE Mathematics Ordinary Level Syllabus 4016		**Common Core State Standards for Mathematics**	
Geometry and Measurement	**SEC Code(s)**	**Geometry**	**SEC Code(s)**
		Congruence	
2.1 Angles, triangles, and polygons Include:		**Experiment with transformations in the plane.**	
• Right, acute, obtuse and reflex angles, complementary and supplementary angles, vertically opposite angles, adjacent angles in a straight line, adjacent angles at a point, interior and exterior angles	701B	**CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	701B, 702B
• angles formed by two parallel lines and a transversal: corresponding angles, alternate angles, interior angles	701B, 706B, 710B	**CO.A.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	716C, 1603D
• properties of triangles and special quadrilaterals	707B, 708B		
• classifying special quadrilaterals on the basis of their properties	708D		
• angles sum of interior and exterior angles of any convex polygon	710B, 711B	**CO.A.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	716D
• properties of regular pentagon, hexagon, octagon and decagon	711B		
• properties of perpendicular bisectors of line segments and angle bisectors	701B, 702B	**CO.A.4** Develop definitions of rotations, reflections, and translations in terms of	716D

• construction of simple geometrical figures from given data (including perpendicular bisectors and angle bisectors) using compasses, ruler, set squares and protractor, where appropriate	301C, 790C	angles, circles, perpendicular lines, parallel lines, and line segments.	
		CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure to another.	716C, 716D, 1603C
2.2 Congruence and similarity Include:		**Understand congruence in terms of rigid motions.**	
• congruent figures and similar figures	704B, 705B	**CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	704D, 716D

• properties of similar polygons:
  • corresponding angles are equal
  • corresponding sides are proportion
	707B, 711B		

• enlargement and reduction of a plane figure by a scale factor	790C		
• scale drawings	790C	**CO.B.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pars of angles are congruent.	704D, 707D, 710D, 716D

• determining whether two triangles are congruent
  • congruent
  • similar
	704C, 705C, 707C		

• ratio of areas of similar plane figures	107C, 306C	**CO.B.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow form the definition of congruence in terms of rigid motions.	704D, 707D, 716D
• ratio of volumes of similar solids	705C		
• solving simple problems involving similarity and congruence	704C, 705C		
2.3 Properties of circles Include:		**Prove geometric theorems.**	
• symmetry properties of circles:	709B, 715B	**CO.C.9** Prove theorems about lines and angles.	702E, 710E, 801E

• equal chords are equidistant from the centre
• the perpendicular bisector of a chord passes through the centre
• tangents from an external point are equal in length
• the line joining an external point to the centre of the circle bisects the angles between the tangents
• angle properties of circles:
  • angle in a semicircle is a right angle
  • angle between tangent and radius of a circle is a right angle
  • angle at the centre is twice the angle at the circumference
  • angles in the same segment are equal
  • angles in opposite segments are supplementary
	709B, 710B	**CO.C.10** Prove theorems about triangles.
		CO.C.11 Prove theorems about parallelograms.
		
		Make geometric constructions.
		CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
		CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
			707E, 801E
			708E, 801E
			
			790C, 1603C
			
			707C, 708C, 709C, 711C

		Similarity, Right Triangles, and Trigonometry	
2.4 Pythagoras' theorem and trigonometry Include: • use of Pythagoras' theorem • determining whether a triangle is right-angled given the lengths of three sides • use of trigonometric ratios (sine, cosine, and tangent) of acute angles to calculate	717D 707D, 717D 707C, 1301C	**Understand similarity in terms of similarity transformations.** **SRT.A.1** Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	705D, 716D

unknown sides and angles in right-angled triangles • extending sine and cosine to obtuse angles • use of the formula $\frac{1}{2}ab\sin C$ for the area of a triangle • use of sine rule and cosine rule for any triangle • problems in 2 and 3 dimensions including those involving angles of elevation and depression and bearings Exclude calculation of the angle between two planes or of the angle between a straight line and a plane. **2.5 Mensuration** Include: • area of parallelogram and trapezium • problems involving perimeter and area of composite plane figures (including triangle and circle) • volume and surface area of cube, cuboid, prism, cylinder, pyramid, cone and sphere • conversion between cm^2 and m^2, and between cm^3 and m^3	707C, 1301C 707C 1304C 1304C, 1390C 306C, 708C, 711C 305C, 306C, 790C 306C, 307C, 712C, 803C 303C, 304C	**SRT.A.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **SRT.A.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **Prove theorems involving similarity.** **SRT.B.4** Prove theorems about triangles. **SRT.B.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **Define trigonometric ratios and solve problems involving right triangles.** **SRT.C.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **SRT.C.7** Explain and use the relationship between the sine and cosine of complementary angles.	705D, 707D, 716D 705D, 710D, 716D 707E, 801E 704C, 705C, 801E 705D, 1301D, 1303D 1301C, 1301D, 1303C

• problems involving volume and surface area of composite solids	306C, 307C, 712C	**SRT.C.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. (★)	713F, 717F, 1301F, 1303F
• arc length and sector area as fractions of the circumference and area of a circle	310B, 709B	**Apply trigonometry to general triangles.**	
• area of a segment	709C	**SRT.D.9** (+) Derive the formula $\frac{1}{2}ab\sin C$ for the area of a triangle by drawing an auxiliary line form a vertex perpendicular to the opposite side.	
• use of radian measure of angle (including conversion between radians and degrees)	1302C	**SRT.D.10** (+) Prove the Laws of Sines and Cosines and use them to solve problems.	
• problems involving the arc length, sector area of a circle and area of a segment	709C	**SRT.D.11** (+) Understand and apply the Law of Sines and Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	
2.6 Coordinate geometry Include:			
• finding the gradient of a straight line given the coordinates of two points on it	804C		
• finding the length of a line segment given the coordinate of its end points	804C		
• interpreting and finding the equation of a straight line graph in the form $y = mx + c$	804C		
• geometric problems involving the use of coordinates	804C		
Exclude: • condition for two lines to be parallel or perpendicular • mid-point of line segment			

• finding the area of quadrilateral given its vertices **2.7 Vectors in two dimensions** Include: • use of notations: $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$, **a**, $\lvert\overrightarrow{AB}\rvert$, and $\lvert\boldsymbol{a}\rvert$ • directed line segments • translation by a vector • position vectors • magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$ • use of sum and difference of two vectors to express given vectors in terms of two coplanar vectors • multiplication of a vector by a scalar • geometric problems involving the use of vectors Exclude: • expressing a vector in terms of a unit vector • mid-point of line segment • solving vector equations with two unknown parameters	 805B 805B 805B 805B 805B 805C 805C 805C		

		Circles	
		Understand and apply theorems about circles. **C.A.1** Prove that all circles are similar. **C.A.2** Identify and describe relationships among inscribed angles, radii, and chords. **C.A.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **C.A.4** (+) Construct a tangent line from a point outside a given circle to the circle. **Find arc lengths and areas of sectors of circles.** **C.B.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	 709E, 801E 709D 707C, 708E, 709C, 709E, 710E, 801E 705E, 709E, 1302B
		Expressing Geometric Properties with Equations	
		Translate between the geometric description and the equation for a conic section. **GPE.A.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square	 717E

		to find the center and radius of a circle given by an equation.	
		GPE.A.2 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	707C, 707E, 709C, 801E
		GPE.A.3 (+) Construct a tangent line from a point outside a given circle to the circle.	
		Use coordinates to prove simple theorems algebraically.	
		GPE.B.4 Use coordinates to prove simple geometric theorems algebraically.	801E, 804E
		GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a parallel line or perpendicular to a given line that passes through a given point)	801E, 804C
		GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	707C, 708C, 711C, 713C, 804C
		1. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. (★)	

		Geometric Measure and Dimension	
		Explain volume formulas and use them to solve problems. **GMD.A.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. **GMD.A.2** (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. **GMD.A.3** Use volume formulas for cylinders, pyramids, cones and spheres to solve problems. (★) **Visualize relationships between two-dimensional and three-dimensional objects.** **GMD.B.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	709E, 712E, 801E, 803E 712C, 713C, 803C, 803F 714D
		Modeling with Geometry	
		Apply geometric concepts in modeling situations. **MG.A.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	

		MG.A.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. **MG.A.3** Use volume formulas for cylinders, pyramids, cones and spheres to solve problems. (★)	

APPENDIX J: SEC CODING NUMBER AND ALGEBRA: SINGAPORE GCE O-LEVEL SECONDARY SYLLABUS AND CCSSM

GCE Mathematics Ordinary Level Syllabus 4016		**Common Core State Standards for Mathematics Grades 9-12**	
Numbers and Algebra	**SEC Code(s)**	**The Real Number System**	**SEC Code(s)**
1.1 Numbers and the four operations Include:		**Extend the properties of exponents to rational exponents.**	
• Primes and prime factorization	111B, 112B	**N.RN.A.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	109D, 110D, 114D
• Finding HCF and LCM, squares, cubes, square roots and cube roots by prime factorization	110C, 111C, 112C, 513C		
• Negative numbers, integers, rational numbers, real numbers and their four operations	109C, 201C, 204C, 204D	**N.RN.A.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.	110C, 218C
• Calculations with the use of a calculator	1601C		
• Representation and ordering of numbers on the number line	114D	**Use properties of rational and irrational numbers.**	
• Use of the symbols $<, >, \leq, \geq$	508D		
• Approximation and estimation (including rounding off numbers to a required number of decimal places or significant figures, estimating the ersults of computation, and concepts of rounding and truncation errors)	105C, 113C	**N.RN.B.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	109D, 190D, 290D
• Examples of very large and very small numbers such as mega/million (10^6), giga/billion (10^9), terra/trillion (10^{12}), micro (10^{-6}), nano (10^{-9}), and pico (10^{-12})	114B		

• Use of standard form $A \times 10^n$, where n is an integer, and $1 \leq A \leq 10$	110C	**Algebra**	
		Seeing Structure in Expressions	
• Positive, negative, zero and fraction indices	607C	**Interpret the structure of expressions.**	
• Laws of indices	608B	**SSE.A.1** Interpret expressions that represent a quantity in terms of its context. (★)	502D, 503D
1.2 Ratio, rate and proportion		a. Interpret parts of an expression, such as terms, factors, and coefficients.	
Include:		b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	
• Ratios involving rational numbers	107C, 109C		
• Writing a ratio in its simplest form	107C, 205C		
• Average rate	314B, 1001B	**SSE.A.2** Use the structure of an expression to identify ways to rewrite it.	516D
• Map scales (distance and area)	107C, 306C, 308C	**Write expressions in equivalent forms to solve problems.**	
• Direct and inverse proportion	210C	**SSE.B.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (★)	512C, 516D, 1504C, 1508C
• Problems involving ratio, rate and proportion	210C, 314C	a. Factor a quadratic expression to reveal the zeroes of the function it defines.	
1.3 Percentage			
Include:			
• Expressing one quantity as a percentage of another	106C, 107C, 217C	b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	
• Comparing two quantities by percentage	106C, 212C		
• Percentages greater than 100%	106C		
• Increasing/decreasing a quantity by a given percentage	106C	c. Use the properties of exponents to transform expressions for exponential functions.	
• Reverse percentages	106C, 217C		

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- Problems involving percentages

1.4 Speed
Include:
- Concepts of speed, uniform speed and average speed
- Conversion of units (e.g. km/h to m/s)
- Problems involving speed, uniform speed and average speed

1.5 Algebraic representation and formulae
Include:
- Using letters to represent numbers
- Interpreting notations:
  - ab as $a \times b$
  - $\frac{a}{b}$ as $a \div b$
  - a^2 as $a \times a, a^3$ as $a \times a \times a, \ldots$
  - $3y$ as $y + y + y$ or $3 \times y$
  - $3(x + y)$ as $3 \times (x + y)$
  - $\frac{3+y}{5}$ as $(3 + y) \div 5$ or $\frac{1}{5} \times (3 + y)$
- Evaluation of algebraic expressions and formulae
- Translation of simple real-world situations into algebraic expressions
- Recognizing and representing number patterns (including finding an algebraic expressions for the *n*th terms)

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106C, 217C

314B, 1001B
303C, 304C
314C, 1001C

502D
516D

503C

502F, 503F, 507F

506E
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SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.
APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeroes and factors of polynomials.
APR.B.2 Know and apply the Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division by *x-a* is *p(a)* = 0 if and only if *(x-a)* is a factor of *p(x)*.
APR.B.3 Identify zeroes of polynomials when suitable factorizations are available, and use the zeroes to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.
APR.C.4 Prove polynomial identities and use them to describe numerical relationships.

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1201E

511C, 511D

611B, 611C

505C, 512C, 1505C

511E, 512D
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1.6 Algebraic manipulation Include:		**APR.C.5** (+) Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle.	
• Addition and subtraction of linear algebraic expressions	509C		
• Simplification of linear algebraic expressions, e.g. $-2(3x-5)+4x$ $\frac{2x}{3}-\frac{3(x-5)}{2}$	503C, 509C, 511C, 515C	**Rewrite rational expressions.**	
• Factorization of linear algebraic expressions of the form ◦ $ax+by$ (where a is a constant) ◦ $ax+bx+kay+kby$ (where a, b, and k are constants)	512C	**APR.D.6** Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x)+\frac{r(x)}{b(x)}$, where *a(x)*, *b(x)*, *q(x)*, and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for more complicated examples, a computer algebra system.	515C, 516C, 1603C
• Expansion of the product of algebraic expressions	503C, 511C		
• Changing the subject of a formula	507C, 516C	**APR.D.7** (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	
• Finding the value of an unknown quantity in a given formula	502C, 507C		
• Recognizing and applying the special products ◦ $(a \pm b)^2 = a^2 \pm 2ab + b^2$ ◦ $a^2-b^2=(a+b)(a-b)$	511C, 512C	**Creating Equations**	
• Factorization of algebraic expressions of the form ◦ $a^2x^2-b^2y^2$ ◦ $a^2 \pm 2ab + b^2$	512C	**Create equations that describe numbers or relationships.** **CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (★)	502C, 507C, 508C

• $ax^2 + bx + c$		**CED.A.2** Create equations in two or more variable to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (★)	502C, 505C
• Multiplication and division of simple algebraic functions, e.g. $\left(\frac{3a}{4b^2}\right)\left(\frac{5ab}{3}\right)$ $\frac{3a}{4} \div \frac{9a^2}{10}$	207C, 208C, 511C, 515C	**CED.A.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	602C, 603C, 690E
• Addition and subtraction of algebraic fractions with linear or quadratic denominator, e.g. $\frac{1}{x-2} + \frac{2}{x-3}$ $\frac{1}{x^2-9} + \frac{2}{x-3}$ $\frac{1}{x-3} + \frac{2}{(x-3)^2}$	206C, 515C	**CED.A.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (★) **Reasoning with Equations and Inequalities** **Understand solving equations as a process of reasoning and explain the reasoning.** **REI.A.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	507C, 516C 507E
1.7 Functions and graphs Include:		**REI.A.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	514C, 515C
• Cartesian coordinates in two dimensions	505D		
• Graph of a set of ordered pairs	505D		
• Linear relationships between two variables (linear functions)	1502C, 1503C		

• The gradient of a linear graph as the ratio of the of the vertical change to the horizontal change (positive and negative gradients)	510D	**Solve equations and inequalities in one variable.**	
• Graphs of linear equations in two unknowns	505C	**REI.B.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	502C, 507C, 508C
• Graphs of quadratic functions and their properties ◦ Positive or negative coefficent of x^2 ◦ Maximum and minimum points ◦ Symmetry	505C, 1504C	**REI.B.4** Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	601C, 601E, 609C
• Sketching the graphs of quadratic functions given in the form ◦ $y = \pm(x-p)^2 + q$ ◦ $y = \pm(x-a)(x-b)$	1504C	b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	
• Graphs of function of the form $y = ax^n$ where $n = -2, -1, 0, 1, 2, 3$, and simple sums of not more than three of these	1508C		
• Graphs of exponential functions $y = ka^x$ where a is a positive integer	1508C		
• Estimation of gradients of curves by drawing tangents	113C, 510C		
1.8 Solutions of equations and inequalities Include:		**Solve systems of equations.**	
• Solving linear equations in one unknown (including fractional coefficients)	507C	**REI.C.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	602E

• Solving simple fractional equations that can be reduced to linear equations, e.g. $$\frac{x}{3}+\frac{x-2}{4}=3$$ $$\frac{3}{x-2}=6$$	507C	**REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **REI.C.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	602C 601C, 602C

• Solving simultaneous linear equations in two unknowns by
  ◦ Subsitution and elimination methods
  ◦ Graphical method
	602C	**REI.C.8** (+) Represent a system of linear equations as a single matrix equation in a vector variable.
		REI.C.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).	

• Solving quadratic equations in one unknown by
  ◦ Factorization
  ◦ Use of formula
  ◦ Completing the square for $y = x^2 + px + q$
  ◦ Graphical methods
	512C, 601C	**Represent and solve equations and inequalities graphically.**
		REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
			505D, 613C

• Solving fractional equations that can be reduced to quadratic equations, e.g. $$\frac{6}{x+4}=x+3$$ $$\frac{1}{x-2}+\frac{2}{x-3}=5$$	601C	**REI.D.11** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational,	1503D, 1505D, 1506D, 1507D, 1508D, 1602C
• Formulating equations to solve problems	502C		

• Solving linear equations in one unknown, and representing the solution set on the number line
 	
1.9 Applications of mathematics in practical solutions
Include:
• Problems derived from practical situations such as
  • Utilities bills
  • Hire-purchase
  • Simple interest and compound interest
  • Money exchange
  • Profit and loss
  • Taxation
 	
• Use of data from tables and charts
• Interpretation and use of graphs in practical situations
• Drawing graphs from given data
• Distance-time and speed-time graphs
 	
Exclude the use of the terms “percentage profit” and “percentage loss”.
	114C, 509C
	
	313F, 401F, 402F, 507F, 590F
	
	901C
	901C
	
	901C
	314C, 901C
		absolute value, exponential, and logarithmic functions.
		REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solutions set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (★)
			
			505C, 603C, 690C

1.10 Set language and notation			
Include:			
• Use of set laguage and the following notation:	1401B		
• Union of A and B — $A \cup B$			
• Intersection A and B — $A \cap B$			
• Number of elements in set A — $n(A)$			
• "…is an element of…" — $\in$			
• "…is not an element of…" — $\notin$			
• Complement of set A — A'			
• The empty set — $\emptyset$			
• Universal set — ξ			
• A is a subset of B — $A \subseteq B$			
• A is a proper subset of B — $A \subset B$			
• A is not a subset of B — $A \nsubseteq B$			
• A is not a proper subset of B — $A \not\subset B$			
• Union and intersection of two sets	1401B		
• Venn diagrams	910C		
Exclude:			
• Use of $n(A \cup B) = n(A) + n(B) - n(A \cap B)$			
• Cases involving three or more sets			
1.11 Matrices			
Include:			
• Display of information in the form of a matrix of any order	605C		
• Interpreting the data in a given matrix	605D		
• Product of a scalar quantitiy and a matrix	605C		

• Problems involing the calucation of the sum and product (where appropriate) of two matrices Exclude: • Matrix representation of geometrical trasnformations • Solving simulataneous linear equations using the inverse matrix method	605C		

APPENDIX K: SEC OFFICIAL CONTENT ANALYSIS: GRADE 8 EQUIVALENT

Percentage of Overall Mathematics Instructional Time

Alignment Overall: 0.131

Coarse Grain Alignment: 0.378

= Not Covered
=< 2.5%
=< 5.0%
=< 7.5%
=>= 7.5%

Administration Year:

Sample Selection: CCSS Gr. 8 | Singapore Sec Stnds Gr. 8

Report By: All Data | All Data

Update

Show Data Tables

Count:	1					1				
Number Sense / Properties / Relationships	2.31	5.72	3.61	0.2	0	0	5.27	0	0	0
Operations	0.3	2.41	0.5	0.2	0	0	5.27	0	0	0
Measurement	0.6	0.7	0	0.1	0.3	0	4.14	0	0	0
Consumer Applications	0	0	0	0	0	0	0	0	0	0
Basic Algebra	1.91	9.53	7.62	3.71	0.4	0	27.71	0	1.03	0
Advanced Algebra	1.81	2.51	2.11	0	2.11	0	5.69	0	2.07	0
Geometric Concepts	3.21	7.92	9.43	7.02	1.1	6.2	16.65	0	0	0
Advanced Geometry	0.8	2.91	0.6	0	0.6	0	0	0	0	0
Data Displays	0.3	2.31	1.5	0.4	0	0	3.1	0	3.1	0
Statistics	0.7	1.91	2.81	0.6	0	0	6.31	0	3.1	0
Probability	0.2	0	0	0	0	0	3.1	0	0	0
Analysis	0	0	0.2	0	0	0	1.03	0	0	0
Trigonometry	0	0	0	0	0	0	0	0	0	0
Special Topics	0	0	0	0	0	3.1	3.1	0	0	0
Functions	2.01	1.91	2.21	0.7	0	0	0	0	0	0
Instructional Technology	0	0	0	0	0	0	0	0	0	0
Student Expectations	I.	II.	III.	IV.	V.	I.	II.	III.	IV.	V.

Student Expectations

I. Memorize Facts, Definitions, Formulas
II. Perform Procedures
III. Demonstrate Understanding
IV. Conjecture, Analyze, Generalize, Prove
V. Solve Non-Routine Problems/Make Connections